Common Errors

Students are sometimes confused by examples such as $\frac{x}{\frac{1}{x}+\frac{1}{y}}$. They may remember that dividing by a nonzero number is equivalent to multiplying by its reciprocal and incorrectly think the above example is equivalent to $x \cdot \left(\frac{x}{1}+\frac{y}{1}\right)$. When reteaching, point out that if this rule is used they must multiply by the reciprocal of the entire divisor:

$$\frac{x}{\frac{1}{x}+\frac{1}{y}} = \frac{x}{\frac{y+x}{xy}} = \frac{x^2y}{y+x}.$$

Guided Practice

Simplify.

1. $\dfrac{2-\frac{1}{2}}{1-\frac{1}{4}}$ 2
2. $\dfrac{1-a}{a^{-1}-1}$ a
3. $\dfrac{\frac{1}{x^2}-\frac{1}{y^2}}{\frac{x}{y}+\frac{y}{x}+2}$ $\frac{y-x}{xy(x+y)}$
4. $\dfrac{k+\frac{1}{k-2}}{\frac{k^2}{k-2}+1}$ $\frac{k-1}{k+2}$

Summarizing the Lesson

In this lesson students learned two methods of simplifying complex fractions. Ask students which of the two methods they prefer.

Using a Calculator

Point out to students that they can keep the number of key presses to a minimum by using the reciprocal key when evaluating the expressions in Exercise 27.

16. $\dfrac{\frac{1}{x}-\frac{1}{y}}{\frac{y}{x}-\frac{x}{y}}$ $\frac{1}{y+x}$

17. $\dfrac{\frac{2}{y+2}-1}{\frac{1}{y+2}+1}$ $-\frac{y}{y+3}$

18. $\dfrac{1+\frac{1}{t-1}}{1-\frac{1}{t+1}}$ $\frac{t+1}{t-1}$

B 19. $\dfrac{\frac{1}{a+1}+\frac{1}{a-1}}{\frac{1}{a+1}-\frac{1}{a-1}}$ $-a$

20. $\dfrac{\frac{1}{x}+\frac{1}{x+1}}{\frac{1}{x}-\frac{1}{x+1}}$ $2x+1$

21. $\dfrac{1+\frac{1}{x-1}}{1+\frac{1}{x^2-1}}$ $\frac{x+1}{x}$

22. $\dfrac{\frac{1}{1-t}-\frac{1}{t}}{\frac{1}{1+t}-\frac{1}{t}}$ $\frac{(2t-1)(t+1)}{t-1}$

23. $\dfrac{\frac{a}{b}-\frac{a-b}{a+b}}{\frac{a}{b}+\frac{a+b}{a-b}}$ $\frac{a-b}{a+b}$

24. $\dfrac{\frac{u+v}{u-v}-\frac{u-v}{u+v}}{\frac{u+v}{u-v}+\frac{u-v}{u+v}}$ $\frac{2uv}{u^2+v^2}$

25. $\dfrac{1-\frac{2-\frac{1}{x}}{x}}{1-\frac{1}{x}}$ $\frac{x-1}{x}$

26. $\dfrac{u+\frac{1}{1+\frac{1}{u}}}{\frac{1}{u+1}}$ u^2+2u

27. Evaluate to three decimal places. A calculator may be helpful.

a. $1+\frac{1}{2}$ 1.500

b. $1+\dfrac{1}{2+\frac{1}{2}}$ 1.400

c. $1+\dfrac{1}{2+\dfrac{1}{2+\frac{1}{2}}}$ 1.417

d. $1+\dfrac{1}{2+\dfrac{1}{2+\dfrac{1}{2+\cdots}}}$ 1.41

(The farther this process is carried out, the closer the results will be to $\sqrt{2} = 1.41421\ldots$.)

In Exercises 28–31, express $\dfrac{f(x+h)-f(x)}{h}$ as a single simplified fraction. (These exercises might be met in calculus.)

Sample $f(x) = \dfrac{1}{1-x}$

Solution

$$\frac{f(x+h)-f(x)}{h} = \frac{\frac{1}{1-(x+h)}-\frac{1}{1-x}}{h}$$

$$= \frac{(1-x)-(1-x-h)}{h(1-x-h)(1-x)}$$

$$= \frac{1}{(1-x-h)(1-x)}$$

C 28. $f(x) = \frac{1}{x}$ $-\frac{1}{x(x+h)}$

29. $f(x) = \frac{1}{x+1}$ $-\frac{1}{(x+1)(x+h+1)}$

30. $f(x) = \frac{1-x}{x}$ $-\frac{1}{x(x+h)}$

31. $f(x) = \frac{1}{x^2}$ $-\frac{2x+h}{x^2(x+h)^2}$

When the numerator or denominator of a complex fraction has powers with negative exponents, you should first rewrite the powers using positive exponents. Then simplify the fraction using either of the methods shown in Example 3.

Example 3 Simplify $\dfrac{a^{-1} - x^{-1}}{a^{-2} - x^{-2}}$.

Solution

Method 1

$$\frac{a^{-1} - x^{-1}}{a^{-2} - x^{-2}} = \left(\frac{1}{a} - \frac{1}{x}\right) \div \left(\frac{1}{a^2} - \frac{1}{x^2}\right)$$

$$= \frac{x - a}{ax} \div \frac{x^2 - a^2}{a^2x^2}$$

$$= \frac{x - a}{ax} \cdot \frac{a^2x^2}{x^2 - a^2}$$

$$= \frac{a^2x^2(x - a)}{ax(x + a)(x - a)}$$

$$= \frac{ax}{x + a}$$

Answer

Method 2

$$\frac{a^{-1} - x^{-1}}{a^{-2} - x^{-2}} = \frac{\left(\frac{1}{a} - \frac{1}{x}\right) \cdot a^2x^2}{\left(\frac{1}{a^2} - \frac{1}{x^2}\right) \cdot a^2x^2}$$

$$= \frac{ax^2 - a^2x}{x^2 - a^2}$$

$$= \frac{ax(x - a)}{(x + a)(x - a)}$$

$$= \frac{ax}{x + a}$$

Answer

Written Exercises

Simplify.

1. $\dfrac{1 - \frac{1}{3}}{\frac{1}{2} - \frac{1}{6}}$ 2

2. $\dfrac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{6}}$ 1

3. $\dfrac{1 - \frac{4}{5}}{\frac{1}{4} - \frac{1}{5}}$ 4

4. $\dfrac{\frac{2}{3} - \frac{5}{6}}{\frac{1}{3} + \frac{2}{9}}$ $-\frac{3}{10}$

5. $\dfrac{x + 1}{1 + \frac{1}{x}}$ x

6. $\dfrac{z - \frac{1}{z}}{1 - \frac{1}{z}}$ $z + 1$

7. $\dfrac{a - b}{a^{-1} - b^{-1}}$ $-ab$

8. $\dfrac{1 - xy^{-1}}{x^{-1} - y^{-1}}$ x

9. $\dfrac{u^{-2} - v^{-2}}{u^{-1} - v^{-1}}$ $\frac{v + u}{uv}$

10. $\dfrac{a^{-2} - b^{-2}}{a^{-1} + b^{-1}}$ $\frac{b - a}{ab}$

11. $\dfrac{\frac{1}{x^2} - \frac{1}{y^2}}{\frac{1}{x^2} + \frac{2}{xy} + \frac{1}{y^2}}$ $\frac{y - x}{y + x}$

12. $\dfrac{\frac{1}{p^2} - \frac{1}{q^2}}{\frac{2}{p^2} - \frac{1}{pq} - \frac{1}{q^2}}$ $\frac{q + p}{2q + p}$

13. $\dfrac{h + h^{-2}}{1 + h^{-1}}$ $\frac{h^2 - h + 1}{h}$

14. $\dfrac{x^{-2} - x^2}{x^{-1} - x}$ $\frac{1 + x^2}{x}$

15. $\dfrac{s^2 - t^{-2}}{s - t^{-1}}$ $\frac{st + 1}{t}$

2. $\dfrac{r^{-3} + s^{-3}}{r^{-1} + s^{-1}}$

Method 1

$$\left(\frac{1}{r^3} + \frac{1}{s^3}\right) \div \left(\frac{1}{r} + \frac{1}{s}\right)$$

$$= \frac{s^3 + r^3}{r^3s^3} \cdot \frac{rs}{s + r}$$

$$= \frac{(s + r)(s^2 - rs + r^2)rs}{r^3s^3(s + r)}$$

$$= \frac{s^2 - rs + r^2}{r^2s^2}$$

Method 2

$$\frac{(r^{-3} + s^{-3}) \cdot r^3s^3}{(r^{-1} + s^{-1}) \cdot r^3s^3}$$

$$= \frac{s^3 + r^3}{r^2s^3 + r^3s^2}$$

$$= \frac{(s + r)(s^2 - rs + r^2)}{r^2s^2(s + r)}$$

$$= \frac{s^2 - rs + r^2}{r^2s^2}$$

3. $\dfrac{1 + \frac{3}{x} + \frac{2}{x^2}}{1 + \frac{2}{x}}$

Method 1

$$\left(1 + \frac{3}{x} + \frac{2}{x^2}\right) \div \left(1 + \frac{2}{x}\right)$$

$$= \frac{x^2 + 3x + 2}{x^2} \cdot \frac{x}{x + 2}$$

$$= \frac{(x + 1)(x + 2)}{x^2} \cdot \frac{x}{x + 2}$$

$$= \frac{x + 1}{x}$$

Method 2

$$\frac{\left(1 + \frac{3}{x} + \frac{2}{x^2}\right) \cdot x^2}{\left(1 + \frac{2}{x}\right) \cdot x^2}$$

$$\frac{x^2 + 3x + 2}{x^2 + 2x} = \frac{(x + 1)(x + 2)}{x(x + 2)} = \frac{x + 1}{x}$$