

# Chapter 11: Sequences and Series

## 11-1 Types of Sequences

**Sequence:** is an ordered set of numbers which could be defined as a function whose domain (x-values) consists of consecutive positive integers and the corresponding value is the range (y-values) of the sequence.

**Term number:** is an ordered set of numbers which could be defined as a function whose domain (x-values) consists of consecutive positive integers.

**Term:** the corresponding value (the range y-value) of the sequence

**Finite:** a sequence with a limited number of terms

**Infinite:** a sequence with an unlimited number of terms

**Arithmetic sequence:** a sequence in which a constant  $d$  (common difference) can be added to each term to get the next term.

**Common difference:** the constant difference, usually denoted as  $d$

**Geometric Sequence:** a sequence in which a constant  $r$  can be multiplied by each term to get the next term

**Common ratio:** the constant ratio, usually denoted by  $r$ .

## 11-2 Arithmetic sequence:

$$t_n = t_1 + (n - 1)d$$

**Arithmetic Mean:** the average between 2 numbers

$$\frac{(a + b)}{2}$$

## 11-3 Geometric Sequence:

$$t_n = t_1 \bullet r^{n-1}$$

**Geometric Mean:** the term between two given terms of a geometric sequence as defined by the following formula:

$$\sqrt{ab}$$

## 11-4 Series and Sigma Notation

**Arithmetic series:** The sum of the terms of an arithmetic sequence.

**Geometric Series:** The sum of the terms of a geometric sequence.

**Sigma:** A series can be written in a shortened form using the Greek letter

$$\sum \quad (\text{Sigma})$$

$$\sum_{n=1}^{50} 2n$$

Summand

$$\sum_{n=1}^{50} 2n$$

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## 11-5 Sums of arithmetic and geometric series

**Sum of an Arithmetic series:**

$$S_n = \frac{n(t_1 + t_n)}{2}, \quad \text{or} \quad S_n = \frac{n}{2} [2t_1 + (n-1)d]$$

**Sum of a geometric series:**

$$S_n = \frac{t_1(1 - r^n)}{1 - r}$$

## 11-6 Infinite Geometric Series

**Theorem:** an infinite geometric series is convergent and has a sum “ $S$ ” if and only if its common ratio,  $r$  meets the following condition:  $|r| < 1$

If our infinite series is convergent ( $|r| < 1$ ), we can calculate its sum by the formula:

$$S = \frac{t_1}{1 - r}$$

## 11-7 Binomial Expansions and Powers of Binomials

**Binomial expansion:**  $(a + b)^n$

You can use Pascal’s Triangle to find the coefficients of the expansion.

## 11-8 The General Binomial Expansion

The Binomial Theorem: for any binomial  $(a + b)$  and any whole number  $n$ , then

$$(a + b)^n = {}_n C_0 a^n + {}_n C_1 a^{n-1} b + {}_n C_2 a^{n-2} b^2 + {}_n C_3 a^{n-3} b^3 + \dots + {}_n C_n b^n$$

**Combinations:**

$$\binom{n}{r} = {}_n C_r = \frac{n!}{(n-r)!r!}$$

**Factorial:**

$$n! = n \cdot (n-1) \cdot (n-2) \cdot \dots \cdot 2 \cdot 1$$

To find the  $r$ th term of a binomial expansion raised to the  $n$ th power, use the following formula:

$$\binom{n}{r-1} a^{(n-r+1)} b^{(r-1)}$$

Which is the same as:

$$\binom{n}{r-1} a^{(n-r+1)} b^{(r-1)}$$

Thanks to my T.A., Jovanna a.k.a. "JT" for creating this review sheet.