Chapter 11: Sequences and Series

11-1 Types of Sequences

**Sequence:** is an ordered set of numbers which could be defined as a function whose domain (x-values) consists of consecutive positive integers and the corresponding value is the range (y-values) of the sequence.

**Term number:** is an ordered set of numbers which could be defined as a function whose domain (x-values) consists of consecutive positive integers.

**Term:** the corresponding value (the range y-value) of the sequence

**Finite:** a sequence with a limited number of terms

**Infinite:** a sequence with an unlimited number of terms

**Arithmetic sequence:** a sequence in which a constant $d$ (common difference) can be added to each term to get the next term.

**Common difference:** the constant difference, usually denoted as $d$

**Geometric Sequence:** a sequence in which a constant $r$ can be multiplied by each term to get the next term

**Common ratio:** the constant ratio, usually denoted by $r$.

11-2 Arithmetic sequence:

$$t_n = t_1 + (n - 1)d$$

**Arithmetic Mean:** the average between 2 numbers

$$\frac{(a + b)}{2}$$

11-3 Geometric Sequence:

$$t_n = t_1 \cdot r^{n-1}$$

**Geometric Mean:** the term between two given terms of a geometric sequence as defined by the following formula:

$$\sqrt{ab}$$
11-4 Series and Sigma Notation

**Arithmetic series:** The sum of the terms of an arithmetic sequence.

**Geometric Series:** The sum of the terms of a geometric sequence.

**Sigma:** A series can be written in a shortened form using the Greek letter \( \sum \) (Sigma)

11-5 Sums of arithmetic and geometric series

**Sum of an Arithmetic series:**

\[
S_n = \frac{n(t_1 + t_n)}{2}, \quad \text{or} \quad S_n = \frac{n}{2} \left[ 2t_1 + (n - 1)d \right]
\]

**Sum of a geometric series:**

\[
S_n = \frac{t_1(1 - r^n)}{1 - r}
\]
11-6 Infinite Geometric Series

**Theorem:** an infinite geometric series is convergent and has a sum “$S$” if and only if its common ratio, $r$ meets the following condition: $|r| < 1$

If our infinite series is convergent ($|r| < 1$), we can calculate its sum by the formula:

$$S = \frac{t_1}{1 - r}$$

11-7 Binomial Expansions and Powers of Binomials

**Binomial expansion:** $(a + b)^n$

You can use Pascal’s Triangle to find the coefficients of the expansion.

11-8 The General Binomial Expansion

The Binomial Theorem: for any binomial $(a + b)$ and any whole number $n$, then

$$(a + b)^n = \sum_{r=0}^{n} \binom{n}{r} a^{n-r} b^r$$

**Combinations:**

$$\binom{n}{r} = \binom{n}{n-r} = \frac{n!}{r!(n-r)!}$$

**Factorial:**

$$n! = n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 2 \cdot 1$$
To find the $r$th term of a binomial expansion raised to the $n$th power, use the following formula:

$$ \binom{n}{r-1} a^{(n-r+1)} b^{(r-1)} $$

Which is the same as:

$$ \binom{n}{r-1} a^{(n-r+1)} b^{(r-1)} $$

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