Chapter 11: Sequences and Series

11-1 Types of Sequences

Sequence: is an ordered set of numbers which could be defined as a function whose domain (x-values) consists of consecutive positive integers and the corresponding value is the range (y-values) of the sequence.

Term number: is an ordered set of numbers which could be defined as a function whose domain (x-values) consists of consecutive positive integers.
Term: the corresponding value (the range y-value) of the sequence
Finite: a sequence with a limited number of terms
Infinite: a sequence with an unlimited number of terms
Arithmetic sequence: a sequence in which a constant ***d*** (common difference) can be added to each term to get the next term.
Common difference: the constant difference, usually denoted as ***d***
Geometric Sequence: a sequence in which a constant ***r*** can be multiplied by each term to get the next term
Common ratio: the constant ratio, usually denoted by ***r***.

11-2 Arithmetic sequence:

$$t_n = t_1 + (n-1)d$$

Arithmetic Mean: the average between 2 numbers

$$\frac{(a+b)}{2}$$

11-3 Geometric Sequence:

$$t_n = t_1 \cdot r^{n-1}$$

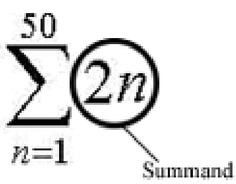
Geometric Mean: the term between two given terms of a geometric sequence as defined by the following formula:

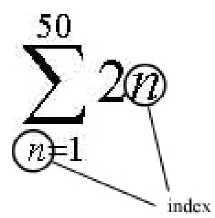
$$\sqrt{ab}$$

11-4 Series and Sigma Notation

Arithmetic series: The sum of the terms of an arithmetic sequence.
Geometric Series: The sum of the terms of a geometric sequence.

Sigma: A series can be written in a shortened form using the Greek letter
$\sum$ (Sigma)





11-5 Sums of arithmetic and geometric series

Sum of an Arithmetic series:

$$s_n = \frac{n(t_1 + t_n)}{2}$$ if you know the first and last term.

or $$s_n = \frac{n}{2}\left[2t_1 + (n-1)d\right]$$ if you know the first term and the common difference.

Sum of a geometric series:

$$s_n = \frac{t_1(1-r^n)}{1-r}$$

11-6 Infinite Geometric Series

Theorem: an infinite geometric series is convergent and has a sum "***S***" if and only if its common ratio, ***r*** meets the following condition: | ***r*** | < 1

If our infinite series is convergent (| ***r*** | < 1), we can calculate its sum by the formula:

$$S = \frac{t_1}{1-r}$$

11-7 Binomial Expansions and Powers of Binomials

Binomial expansion: $(a+b)^n$

You can use Pascal's Triangle to find the coefficients of the expansion.

11-8 The General Binomial Expansion

The Binomial Theorem: for any binomial (a + b) and any whole number ***n***, then

$(a+b)^n$ =

$${}_nC_0a^n +{}_n C_1a^{n-1}b+{}_n C_2a^{n-2}b^2 +{}_n C_3a^{n-3}b^3 +...+{}_n C_nb^n$$

Combinations:

$$\binom{n}{r} ={}_n C_r = \frac{n!}{(n-r)!r!}$$

Factorial:

$$n! = n\cdot(n-1)\cdot(n-2)\cdot...\cdot 2\cdot 1$$

To find the ***r***th term of a binomial expansion raised to the ***n***th power, use the following formula:

$$\binom{n}{r-1} a^{(n-r+1)} b^{(r-1)}$$

Which is the same as:

$$\left({}_{n}C_{r-1}\right) a^{(n-r+1)} b^{(r-1)}$$

	Arithmetic	**Note:**
Sequence	$t_n = t_1 + (n-1)d$	Arithmetic Sequence
Series (Find the sum)	$s_n = \dfrac{n(t_1 + t_n)}{2}$	When you know the first and last term.
	$s_n = \dfrac{n}{2}\left[2t_1 + (n-1)d\right]$	When you know the first term and the common difference.
	Geometric	**Note:**
Sequence	$t_n = t_1 \bullet r^{n-1}$	Geometric Sequence
Series (Find the sum)	$s_n = \dfrac{t_1(1-r^n)}{1-r}$	A finite Geometric Series (a limited number of terms, or Partial Sum)
	$S = \dfrac{t_1}{1-r}$	An **infinite** Geometric Series, if our infinite series is convergent (| ***r*** | < 1)