

Chapter 12

12-1 Angles and Degree Measure

Initial Side: the ray that you start with when creating an angle.

Terminal Side: the ray where the rotation ends.

Standard Position: an angle where the initial side starts on the positive x-axis.

Positive Angles: are created by a counterclockwise rotation

Negative Angles: are created by a clockwise rotation

Quadrantal Angle: when the terminal side lies on the x-axis or y-axis.

Coterminal: when the terminal sides of two standard position angles coincide (are the same line)

(The angle with coterminal sides) $+n \cdot 360^\circ$

One minute ($1'$) can be divided into 60 seconds ($60''$), or $1' = 60''$. Use the following summary for degrees, minutes, and seconds:

$$1' = \left(\frac{1}{60}\right)^\circ \quad 1'' = \left(\frac{1}{60}\right)' = \left(\frac{1}{3600}\right)^\circ$$

12-2 Trigonometric Functions of Acute Angles

The six trigonometric Ratios in a Right Triangle involving one angle and two sides.

$$\begin{array}{ll} \sin \theta = \frac{y}{r} = \frac{\textit{Opposite}}{\textit{Hypotenuse}} & \csc \theta = \frac{r}{y} = \frac{\textit{Hypotenuse}}{\textit{Opposite}} \\ \cos \theta = \frac{x}{r} = \frac{\textit{Adjacent}}{\textit{Hypotenuse}} & \sec \theta = \frac{r}{x} = \frac{\textit{Hypotenuse}}{\textit{Adjacent}} \\ \tan \theta = \frac{y}{x} = \frac{\textit{Opposite}}{\textit{Adjacent}} & \cot \theta = \frac{x}{y} = \frac{\textit{Adjacent}}{\textit{Opposite}} \end{array}$$

Memory Devices for Sin, Cos, & Tan:

- 1) SOH CAH TOA
- 2) *O*scar *H*ad *A* *H*eaP *O*f Apples (Sin, Cos, Tan)
- 3) *S*addle *O*ur *H*orses, *C*antor *A*way *H*appily *T*owards *O*ther Adventures

Memory Devices for Csc, Sec, & Cot:

- 1) *C*alculators *H*elps *O*ur *S*mart *H*eads *A*t *C*alculating *A*lgebraic *O*perations.
- 2) Csc (at Christmas Santa Claus says **HO, HO, HO**)
- 3) Sec (If you think of the game "Pin The Tail on the Donkey", then Secant is the funniest one: **HA, HA, HA**)
- 4) Cot (Calypso music has an "A O" in it, sing it now...
"A O, A O daylight come and we want to go home".)

Trigonometric Identity: is an equation involving trigonometric functions of an angle θ that is true for all values of θ .

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

Complimentary Angles: are angles whose sum is 90°

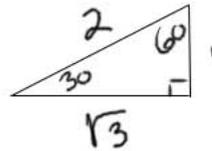
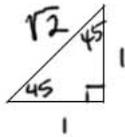
Co-function Identities

$$\sin \theta = \cos(90 - \theta) \quad \cos \theta = \sin(90 - \theta)$$

$$\tan \theta = \cot(90 - \theta) \quad \cot \theta = \tan(90 - \theta)$$

$$\sec \theta = \csc(90 - \theta) \quad \csc \theta = \sec(90 - \theta)$$

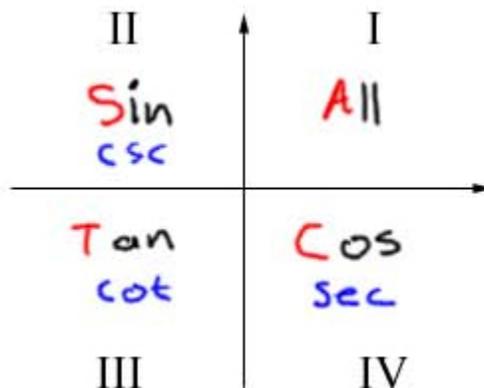
Special Right Triangles from Geometry



θ	$\sin \theta$	$\cos \theta$	$\tan \theta$	$\csc \theta$	$\sec \theta$	$\cot \theta$
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$

12-3 Trigonometric Functions of General Angles

The following Trigonometric Functions have positive values in the Quadrants shown.



All Students Take Calculus

Reference Angle: is an acute angle formed by the terminal side and the x-axis.

12-4 Values of Trigonometric Functions

This section covered using your calculator to find the values of Trig. Functions.

12-5 Solving Right Triangles

Solving the triangle: is finding the measures of all sides and all angles in a triangle.

Angle of elevation: is found by the line of sight looking horizontally out to the horizon and then looking up at an object.

Angle of depression: is found by the line of sight looking horizontally out to the horizon and then looking down at an object.

12-6 The Law of Cosines

The Law of Cosines is used to solve triangles that are not right triangles.

$$\begin{aligned}a^2 &= b^2 + c^2 - 2bc \cos A \\b^2 &= a^2 + c^2 - 2ac \cos B \\c^2 &= a^2 + b^2 - 2ab \cos C\end{aligned}$$

12-7 The Laws of Sines

The Law of Sines is used to solve triangles that are not right triangles.

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

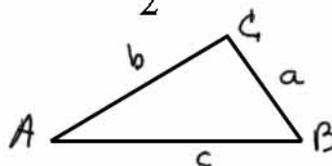
12-9 Areas of Triangles: Law of Sines, Hero's Formula

The Law of Sines (Triangle Area Formula): is best suited to triangles where you know the measures of two sides and the included angle (SAS) and wish to calculate the area. The formulas are:

$$\text{Area} = \frac{1}{2}bc \sin A$$

$$\text{Area} = \frac{1}{2}ac \sin B$$

$$\text{Area} = \frac{1}{2}ab \sin C$$



Hero's Formula can be used if you know all three sides of the triangle. This formula is a two step process:

- 1) Calculate a variable called s (**half the sum** of all three sides)
- 2) Use s and the three sides (a, b, c) to calculate the Area

Here are the two parts of the formula:

$$s = \frac{1}{2}(a + b + c)$$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$