# Chapter 12

## 12-1 Angles and Degree Measure

**Initial Side:** the ray that you start with when creating an angle.
**Terminal Side:** the ray where the rotation ends.
**Standard Position:** an angle where the initial side starts on the positive x-axis.
**Positive Angles:** are created by a counterclockwise rotation
**Negative Angles:** are created by a clockwise rotation
**Quadrantal Angle:** when the terminal side lies on the x-axis or y-axis.
**Coterminal:** when the terminal sides of two standard position angels coincide (are the same line)

**(The angle with conterminal sides)** $+n \bullet 360°$

One minute (1') can be divided into 60 seconds (60''), or 1'=60''. Use the following summary for degrees, minutes, and seconds:

$$1' = \left(\frac{1}{60}\right)^{\circ} \quad 1'' = \left(\frac{1}{60}\right)' = \left(\frac{1}{3600}\right)^{\circ}$$

## 12-2 Trigonometric Functions of Acute Angles

The six trigonometric Ratios in a Right Triangle involving one angle and two sides.

$$\sin\theta = \frac{y}{r} = \frac{Opposite}{Hypotenuse} \qquad \csc\theta = \frac{r}{y} = \frac{Hypotenuse}{Opposite}$$

$$\cos\theta = \frac{x}{r} = \frac{Adjacent}{Hypotenuse} \qquad \sec\theta = \frac{r}{x} = \frac{Hypotenuse}{Adjacent}$$

$$\tan\theta = \frac{y}{x} = \frac{Opposite}{Adjacent} \qquad \cot\theta = \frac{x}{y} = \frac{Adjacent}{Opposite}$$

Memory Devices for Sin, Cos, & Tan:
1) SOH CAH TOA
2) ***O***scar ***H***ad ***A*** ***H***eap ***O***f ***A***pples
3) ***S***ome ***O***ld ***H***ags ***C***an't ***A***lways ***H***ide ***T***heir ***O***ld ***A***ge (not P.C.)
4) ***S***addle ***O***ur ***H***orses, ***C***antor ***A***way ***H***appily ***T***owards ***O***ther ***A***dventures

Memory Devices for Csc, Sec, & Cot:
1) ***C***lausen ***H***elps ***O***ur ***S***mart ***H***eads ***A***t ***C***alculating ***A***lgebraic ***O***perations.
2) Csc (at ***C***hristmas ***S***anta ***C***laus says **HO, HO, HO**)
3) Sec (Secant is the funniest one: **HA, HA, HA**)
4) Cot (Calypso music always has an **A O** in it, sing it now...
"A O, A O daylight come and we want to go home".)

**Trigonometric Identity:** is an equation involving trigonometric functions of an angle $\theta$ that is true for all values of $\theta$.

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

**Complimentary Angles:** are angles whose sum is $90°$

**Co-function Identities**

$$\sin\theta = \cos(90-\theta) \qquad \cos\theta = \sin(90-\theta)$$

$$\tan\theta = \cot(90-\theta) \qquad \cot\theta = \tan(90-\theta)$$

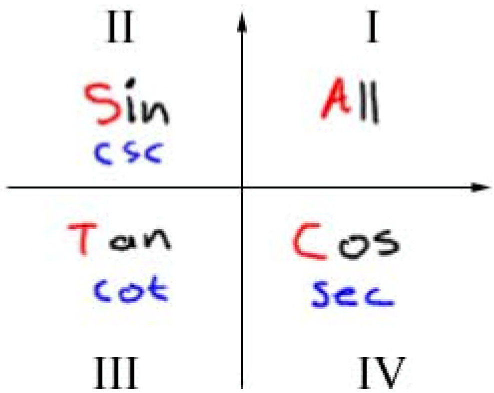
$$\sec\theta = \csc(90-\theta) \qquad \csc\theta = \sec(90-\theta)$$

**Special Right Triangles from Geometry**


| $\theta$ | $\sin\theta$ | $\cos\theta$ | $\tan\theta$ | $\csc\theta$ | $\sec\theta$ | $\cot\theta$ |
|---|---|---|---|---|---|---|
| $30°$ | $\frac{1}{2}$ | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{3}}{3}$ | $2$ | $\frac{2\sqrt{3}}{3}$ | $\sqrt{3}$ |
| $45°$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{2}}{2}$ | $1$ | $\sqrt{2}$ | $\sqrt{2}$ | $1$ |
| $60°$ | $\frac{\sqrt{3}}{2}$ | $\frac{1}{2}$ | $\sqrt{3}$ | $\frac{2\sqrt{3}}{3}$ | $2$ | $\frac{\sqrt{3}}{3}$ |

## 12-3 Trigonometric Functions of General Angles

The following Trigonometric Functions have positive values in the Quadrants shown.




Here's a saying that can help you remember this:
***A**ll **S**candinavians **T**an **C**arefully*

(it's OK, I'm Scandinavian, so I'm not Culture Slamming)

**Don't forget that the reciprocals of these functions are positive too.**

**Reference Angle:** is an acute angle formed by the terminal side and the x-axis.

## 12-4 Values of Trigonometric Functions

This section covered using your calculator to find the values of Trig. Functions.

## 12-5 Solving Right Triangles

**Solving the triangle:** is finding the measures of all sides and all angles in a triangle.
**Angle of elevation:** is found by the line of sight looking horizontally out to the horizon and then looking up at an object.
**Angle of depression:** is found by the line of sight looking horizontally out to the horizon and then looking down at an object.

## 12-6 The Law of Cosines

The Law of Cosines is used to solve triangles that are not right triangles.

$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$b^2 = a^2 + c^2 - 2ac \cos B$$
$$c^2 = a^2 + b^2 - 2ab \cos C$$

## 12-7 The Laws of Sines

The Law of Sines is used to solve triangles that are not right triangles.

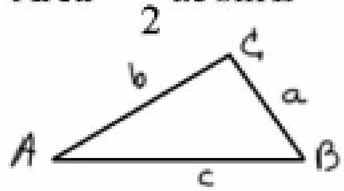
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

## 12-9 Areas of Triangles: Law of Sines, Hero's Formula

**The Law of Sines (Triangle Area Formula): is best suited to triangles where you know the measures of two sides and the included angle (SAS) and wish to calculate the area. The formulas are:**

$$\text{Area} = \frac{1}{2} bc \sin A \qquad \text{Area} = \frac{1}{2} ac \sin B$$

$$\text{Area} = \frac{1}{2} ab \sin C$$




**Hero's Formula** can be used if you know all three sides of the triangle. This formula is a two step process:

1) Calculate a variable called ***s*** (**half the sum** of all three sides)
2) Use ***s*** and the three sides (***a***, ***b***, ***c***) to calculate the Area

Here are the two parts of the formula:

$$s = \frac{1}{2}(a + b + c)$$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

Thanks to my T.A., Jovanna "JT" for making this review sheet.