Chapter 12

12-1 Angles and Degree Measure

Initial Side: the ray that you start with when creating an angle.
Terminal Side: the ray where the rotation ends.
Standard Position: an angle where the initial side starts on the positive x-axis.
Positive Angles: are created by a counterclockwise rotation
Negative Angles: are created by a clockwise rotation
Quadrantal Angle: when the terminal side lies on the x-axis or y-axis.
Coterminal: when the terminal sides of two standard position angles coincide (are the same line). **Coterminal angles differ by a multiple of 360 degrees.**

(The angle with conterminal sides) $+n \cdot 360°$

One minute (1') can be divided into 60 seconds (60''), or 1'=60''. Use the following summary for degrees, minutes, and seconds:

$$1' = \left(\frac{1}{60}\right)^{\circ} \quad 1'' = \left(\frac{1}{60}\right)' = \left(\frac{1}{3600}\right)^{\circ}$$

12-2 Trigonometric Functions of Acute Angles

The six trigonometric Ratios of a Right Triangle involving one angle and two sides.

$$\sin\theta = \frac{y}{r} = \frac{Opposite}{Hypotenuse} \qquad \csc\theta = \frac{r}{y} = \frac{Hypotenuse}{Opposite}$$

$$\cos\theta = \frac{x}{r} = \frac{Adjacent}{Hypotenuse} \qquad \sec\theta = \frac{r}{x} = \frac{Hypotenuse}{Adjacent}$$

$$\tan\theta = \frac{y}{x} = \frac{Opposite}{Adjacent} \qquad \cot\theta = \frac{x}{y} = \frac{Adjacent}{Opposite}$$

Memory Devices for Sin, Cos, & Tan:
1) SOH CAH TOA
2) ***O***scar ***H***ad ***A*** ***H***eap ***O***f ***A***pples
3) ***S***ome ***O***ld ***H***ags ***C***an't ***A***lways **H**ide ***T***heir ***O***ld ***A***ge (not P.C.)
4) ***S***addle ***O***ur ***H***orses, ***C***antor ***A***way ***H***appily ***T***owards ***O***ther ***A***dventures

Memory Devices for Csc, Sec, & Cot:
1) ***C***lausen ***H***elps ***O***ur ***S***mart ***H***eads ***A***t ***C***alculating ***A***lgebraic ***O***perations.
2) Csc (at **C**hristmas **S**anta ***C***laus says **HO, HO, HO**)
3) Sec (Secant is the funniest one: **HA, HA, HA**)
4) Cot (Calypso music always has an **A O** in it, sing it now... "A O, A O daylight come and we want to go home".)

Trigonometric Identity: is an equation involving trigonometric functions of an angle θ that is true for all values of θ.

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

Complimentary Angles: are angles whose sum is $90°$

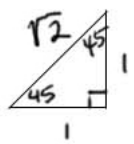
Co-function Identities

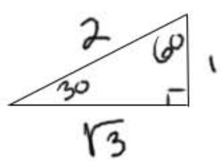
$$\sin\theta = \cos(90-\theta) \qquad \cos\theta = \sin(90-\theta)$$

$$\tan\theta = \cot(90-\theta) \qquad \cot\theta = \tan(90-\theta)$$

$$\sec\theta = \csc(90-\theta) \qquad \csc\theta = \sec(90-\theta)$$

Special Right Triangles from Geometry





θ	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
$30°$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
$45°$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
$60°$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$

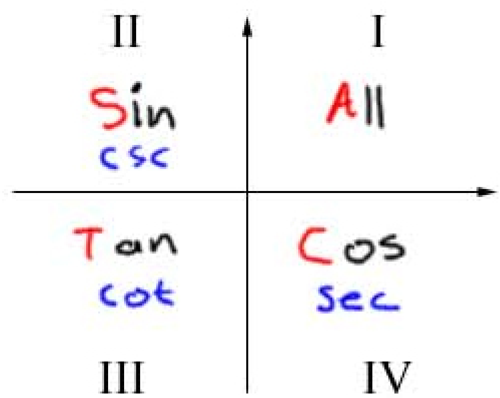
Another way to remember some Trig. values:

Sin 0	$= \frac{\sqrt{0}}{2} = 0$	Cos 90
Sin 30	$= \frac{\sqrt{1}}{2} = \frac{1}{2}$	Cos 60
Sin 45	$= \frac{\sqrt{2}}{2}$	Cos 45
Sin 60	$= \frac{\sqrt{3}}{2}$	Cos 30
Sin 90	$= \frac{\sqrt{4}}{2} = \frac{2}{2} = 1$	Cos 0

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

12-3 Trigonometric Functions of General Angles

The following Trigonometric Functions have positive values in the Quadrants shown.

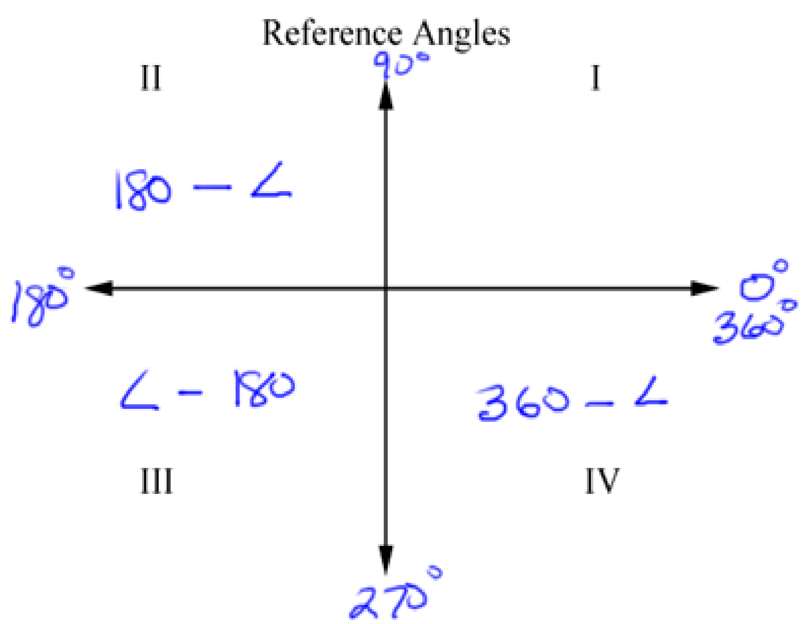


Here's a saying that can help you remember this:
***A**ll **S**candinavians **T**an **C**arefully*

(it's OK, I'm Scandinavian, so I'm not Culture Slamming)

Don't forget that the reciprocals of these functions are positive too.

Reference Angle: is an acute angle formed by the terminal side and the x-axis. Use the following chart to find the reference angle in quadrants II, III or IV.



12-4 Values of Trigonometric Functions

This section covered using your calculator to find the values of Trig. Functions.

12-5 Solving Right Triangles

Solving the triangle: is finding the measures of all sides and all angles in a triangle.
Angle of elevation: is found by the line of sight looking horizontally out to the horizon and then looking up at an object.
Angle of depression: is found by the line of sight looking horizontally out to the horizon and then looking down at an object.

12-6 The Law of Cosines

The Law of Cosines is used to solve triangles that are not right triangles.

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

12-7 The Laws of Sines

The Law of Sines is used to solve triangles that are not right triangles.

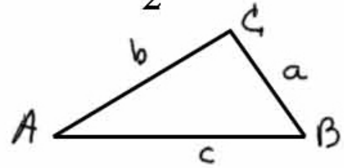
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

12-9 Areas of Triangles: Law of Sines, Hero's Formula

The Law of Sines (Triangle Area Formula): is best suited to triangles where you know the measures of two sides and the included angle (SAS) and wish to calculate the area. The formulas are:

$$\text{Area} = \frac{1}{2}bc\sin A \qquad \text{Area} = \frac{1}{2}ac\sin B$$

$$\text{Area} = \frac{1}{2}ab\sin C$$



Hero's Formula can be used if you know all three sides of the triangle. This formula is a two step process:

1) Calculate a variable called ***s*** (**half the sum** of all three sides)
2) Use ***s*** and the three sides (***a***, ***b***, ***c***) to calculate the Area

Here are the two parts of the formula:

$$s = \frac{1}{2}(a + b + c)$$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$