# Chapter 16 Matrices and Determinants

## 16-1 Definition of terms

**Matrix:** is a rectangular array of numbers enclosed by square brackets (plural of matrix is matrices).

**Elements:** objects in a matrix.

**Dimensions:** are determined by the number of rows (horizontal) and columns (vertical).

$m \times n$ (Rows before columns: RC Cola)

**Square matrix:** is a matrix that has the same number of rows and columns**.**

**Zero Matrix:** is a matrix where all the elements are zeros.

## 16-5 Calculating Determinants Using the Definition

**Determinant:** only square matrices have determinants. This is a real number associated with square matrices (definitions to follow).

**det A: |A|**

**Order:** the number of elements in any row or column (since the number of rows and the columns is the same)

The determinant of matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is denoted by $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$

The det A is the same as |A|.

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - cb$$

The determinant of a 3 x 3 matrix B, det B is defined as follows:

$$\det B = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = aei + bfg + cdh - gec - hfa - idb$$

# 16-7 Determinants: Expansion by Minors

**Minor of an element:** The determinant formed by “deleting” or “hiding” the row and column that contains the elements.

To Calculate the Determinant using Expansion by Minors

1) Select a row or column to use.
2) Multiply each element in this row or column by the determinant of its minor matrix.
3) Add the row and column numbers for each element.
   If the sum is odd, multiply the product obtained in (2) by -1.
   Or use the matrix of alternating signs listed below.
4) Add the products to find the value of the determinant.

$$\begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = +a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

# 16-9 Part 1 Cramer’s Rule (2 Eqs. & 2 Vars.)

**Cramer’s Rule:** used to solve systems equations.

$$ax + by = c$$
$$dx + ey = f$$

$D_d = \begin{vmatrix} a & b \\ d & e \end{vmatrix}$ This determinant will give us the denominator.

$D_x = \begin{vmatrix} c & b \\ f & e \end{vmatrix}$ This determinant will give us the numerator of $x$.

$D_y = \begin{vmatrix} a & c \\ d & f \end{vmatrix}$ This determinant will give us the numerator of $y$.

$$x = \frac{D_x}{D_d} = \frac{\begin{vmatrix} c & b \\ f & e \end{vmatrix}}{\begin{vmatrix} a & b \\ d & e \end{vmatrix}}$$

$$y = \frac{D_y}{D_d} = \frac{\begin{vmatrix} a & c \\ d & f \end{vmatrix}}{\begin{vmatrix} a & b \\ d & e \end{vmatrix}}$$

**Inconsistent or Dependent**

If $D_d = 0$ AND $D_y \neq 0$ the equations are inconsistent and their graphs are parallel.

If $D_d = 0$ AND $D_y = 0$ the equations are consistent and dependent (graphs are the same line).

# 16-9 Cramer's Rule Part 2 (3 Eqs. & 3 Vars.)

Solve Systems with 3 equations and 3 variables

$$ax + by + cz = d$$
$$ex + fy + gz = h$$
$$ix + jy + kz = l$$

We need to find four matrices and calculate their determinants...

$$D_d = \begin{vmatrix} a & b & c \\ e & f & g \\ i & j & k \end{vmatrix}$$

$$D_x = \begin{vmatrix} d & b & c \\ h & f & g \\ l & j & k \end{vmatrix}$$

$$D_y = \begin{vmatrix} a & d & c \\ e & h & g \\ i & l & k \end{vmatrix}$$

$$D_z = \begin{vmatrix} a & b & d \\ e & f & h \\ i & j & l \end{vmatrix}$$

$$x = \frac{D_x}{D_d} \quad y = \frac{D_y}{D_d} \quad z = \frac{D_z}{D_d}$$