

Reminder (properties of exponents):

$$a^m \cdot a^n = a^{m+n}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$(ab)^m = a^m b^m$$

$$(a^m)^n = a^{mn}$$

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

$$a^0 = 1$$

$$a^{-m} = \frac{1}{a^m}$$

If m and n are integers with $n > 0$, and b is a positive real number, then

$$b^{\frac{m}{n}} = \left(\sqrt[n]{b}\right)^m = \sqrt[n]{b^m}$$

Ex 1 Simplify: $27^{\frac{2}{3}}$

Solution 1: $27^{\frac{2}{3}} = (\sqrt[3]{27})^2 = 3^2 = 9$

Solution 2: $27^{\frac{2}{3}} = \sqrt[3]{27^2} = \sqrt[3]{729} = 9$

EX 2a) $64^{-\frac{3}{2}}$

$$\frac{1}{64^{\frac{3}{2}}}$$

$$\frac{1}{(\sqrt{64})^3}$$

$$\frac{1}{8^3}$$

$$\frac{1}{512}$$

EX 2b)

$$16^{2.5}$$

$$16^{\left(\frac{5}{2}\right)}$$

$$(\sqrt{16})^5$$

$$4^5$$

$$1024$$

EX 3a) Write in exponential form:

$$\sqrt[3]{\frac{x^7 y^5}{z^2}} = \left(\frac{x^7 y^5}{z^2} \right)^{\frac{1}{3}}$$

$$\frac{x^{\frac{7}{3}} y^{\frac{5}{3}}}{z^{\frac{2}{3}}} =$$

$$x^{\frac{7}{3}} y^{\frac{5}{3}} z^{-\frac{2}{3}}$$

EX 3b Simplify: $\left(\frac{1}{\sqrt[3]{9}} \right)^{-\frac{3}{2}}$

$$\left(\sqrt[3]{9} \right)^{\frac{3}{2}} \rightarrow \left(9^{\frac{1}{3}} \right)^{\frac{3}{2}} \rightarrow 9^{\frac{1}{2}} \rightarrow \sqrt{9} \rightarrow \textcircled{3}$$

EX4 Write in exponential form
and simplest radical form:

$$\sqrt{8} \cdot \sqrt[3]{4}$$

$$8^{\frac{1}{2}} \cdot 4^{\frac{1}{3}}$$

$$(2^3)^{\frac{1}{2}} \cdot (2^2)^{\frac{1}{3}}$$

$$2^{\frac{3}{2}} \cdot 2^{\frac{2}{3}}$$

$$2^{\frac{3}{2} + \frac{2}{3}} \rightarrow 2^{\frac{9}{6} + \frac{4}{6}} \rightarrow 2^{\frac{13}{6}} \rightarrow 2^{2\frac{1}{6}}$$

$$2^2 \cdot 2^{\frac{1}{6}} \rightarrow 4 \sqrt[6]{2}$$

$$\text{EX 5a) Solve: } (x-1)^{\frac{3}{2}} = 8$$

$$\left[(x-1)^{\frac{3}{2}} \right]^{\frac{2}{3}} = 8^{\frac{2}{3}}$$

$$x-1 = 4$$

$$\boxed{x = 5}$$

$$\text{EX 5b) } \frac{5x^{-\frac{1}{3}}}{5} = \frac{20}{5}$$

$$\left(x^{-\frac{1}{3}} \right)^{-3} = (4)^{-3}$$

$$x^1 = \frac{1}{4^3}$$

$$\boxed{x = \frac{1}{64}}$$