10-3 Composition and Inverses of Functions Page 463

Alg. 2 Standard 24.0 Students solve problems involving functional concepts, such as **composition, defining the inverse function** and performing arithmetic operations on functions.

Objective: 1) To find the composite of two given functions, and 2) to find the inverse of a given function.

If ***f*** and ***g*** are any two functions such that the range of ***g*** is in the domain of ***f***. The **composition** of ***f*** and ***g*** is the function denoted by ***f(g(x))***. $f \circ g$

Ex 1) If $f(x) = x^2$ and $g(x) = x + 2$
find $f(g(3))$.

$$3 \longrightarrow \underset{\boxed{x+2}}{g(x)} \xrightarrow{5} \underset{\boxed{x^2}}{f(x)} \longrightarrow 25$$

$f(g(3)) = 25$

__Ex 2__ if $f(x) = \frac{x+4}{2}$ and $g(x) = 2x - 4$

2a) find $f(g(2))$

$$2 \longrightarrow \overset{g(x)}{\boxed{2x-4}} \xrightarrow{0} \overset{f(x)}{\boxed{\frac{x+4}{2}}} \longrightarrow 2$$

$f(g(2)) = 2$

2b) find $f(g(x))$

$$x \xrightarrow{\quad} \underset{2x-4}{g(x)} \xrightarrow{(2x-4)} \underset{\frac{x+4}{2}}{f(x)} \xrightarrow{\quad} \left[\frac{(2x-4)+4}{2}\right]$$

$$f(g(x)) = x$$

2c) find $g(f(x))$

$$x \xrightarrow{\quad} \underset{\frac{x+4}{2}}{f(x)} \xrightarrow{\frac{x+4}{2}} \underset{2x-4}{g(x)} \xrightarrow{\quad} 2\left(\frac{x+4}{2}\right)-4$$

$$g(f(x)) = x$$

Inverse Functions

The functions *f* and *g* are inverse functions if

1) $f(g(x)) = x$ for all x values in the domain of *g* &
2) $g(f(x)) = x$ for all x values in the domain of *f*.

The inverse of the function f is denoted by f^{-1} (f inverse).

Horizontal Line Test
A function has an inverse if and only if every horizontal line intersects the graph of the function in ***at most*** one point.

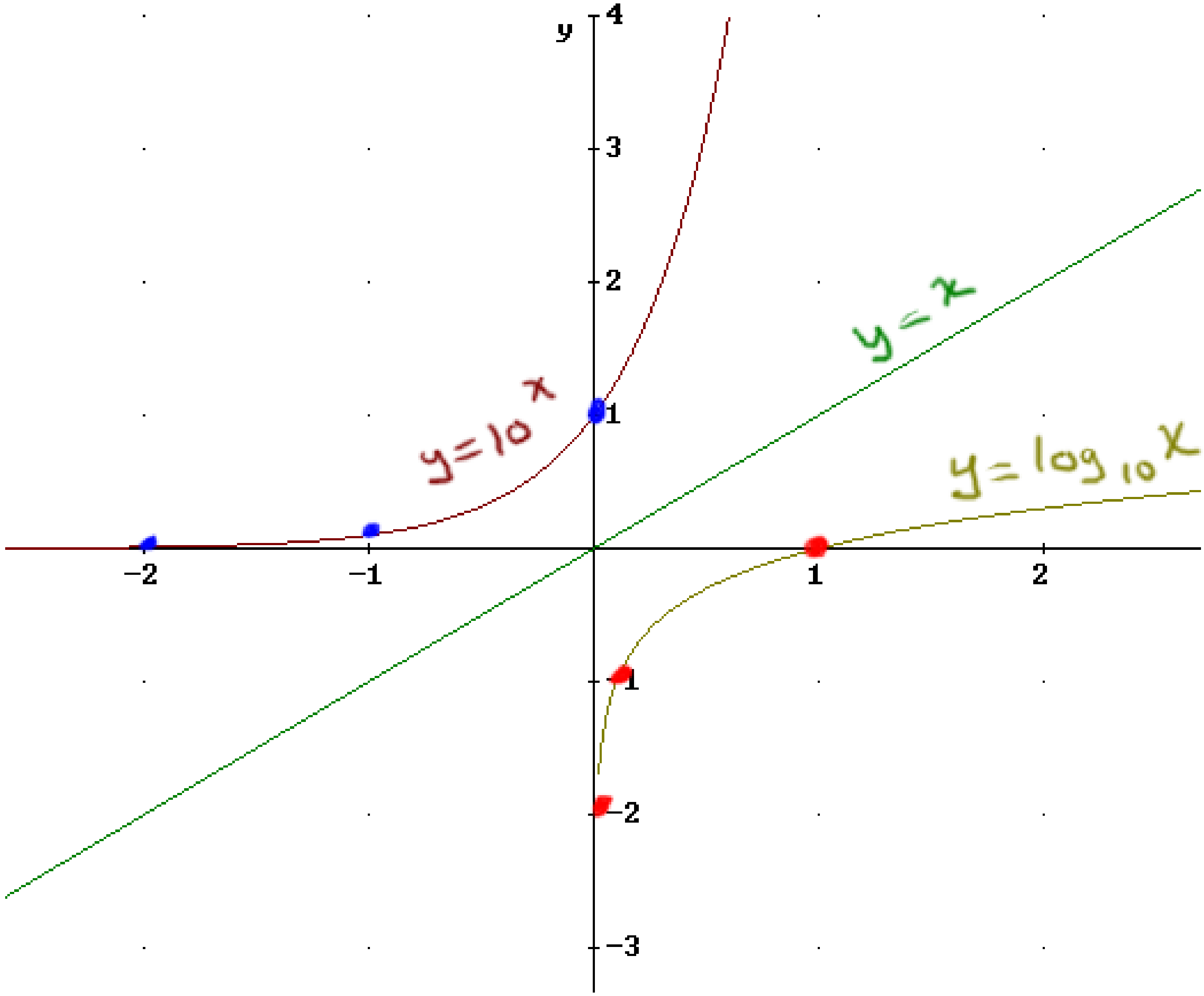
Inverse functions are mirror images of each other with respect to the line y = x. This means that every point (a, b) on one graph corresponds to the point (b, a) on the other graph.

Ex 3) Consider the following inverse functions, and graph them.

$y = 10^x$ and $y = \log_{10} x$

x	y
-2	1/100
-1	1/10
0	1
1	10
2	100

x	y
$\frac{1}{100}$	-2
$\frac{1}{10}$	-1
1	0
10	1
100	2



The inverse of the exponential function with base 10 is called the logarithmic function with base 10.

Ex 4) Let $f(x)=x^3-1$,
find the inverse function $f^{-1}(x)$

Strategy: 1) Replace *f(x)* with *y*
2) Interchange *x* and *y*
3) Solve for *y*
4) Replace y with $f^{-1}(x)$

$$y = x^3 - 1$$

$$x = y^3 - 1$$

$$+1 \qquad +1$$

$$x + 1 = y^3$$

$$y^3 = x + 1$$

$$\sqrt[3]{y^3} = \sqrt[3]{x+1}$$

$$y = \sqrt[3]{x+1}$$

$$f^{-1}(x) = \sqrt[3]{x+1}$$