

Definition of Logarithm

If b and N are positive numbers ($b \neq 1$)

$$\log_b N = k \text{ if and only if } b^k = N$$

↑ ↑
 base exponent
 ↓ ↓
 number

Rewrite the following exponential functions in logarithmic form, and rewrite the following logarithms in exponential form.

Exponential Form	Logarithmic Form
$2^3 = 8$	$\log_2 8 = 3$
$2^4 = 16$	$\log_2 16 = 4$
$2^0 = 1$	$\log_2 1 = 0$
$2^{-1} = \frac{1}{2}$	$\log_2 \frac{1}{2} = -1$
$2^k = N$	$\log_2 N = k$

Ex 1) Rewrite each equation in exponential form:

$$1a) \log_6 36 = 2 \rightarrow 6^2 = 36$$

$$1b) \log_2 2 = 1 \rightarrow 2^1 = 2$$

$$1c) \log_{10}(0.001) = -3 \rightarrow 10^{-3} = 0.001$$

Ex 2) Rewrite the following in logarithmic form:

$$2a) 6^0 = 1 \rightarrow \log_6 1 = 0$$

$$2b) 8^{-\frac{2}{3}} = \frac{1}{4} \rightarrow \log_8 \frac{1}{4} = -\frac{2}{3}$$

$$2c) 5^{\frac{3}{2}} = 5\sqrt{5} \rightarrow \log_5 5\sqrt{5} = \frac{3}{2}$$

Ex 3) Simplify each logarithm:

3a) $\log_5 25$

$$\log_5 25 = x$$

$$5^x = 25$$
$$5^x = \cancel{5^2}$$

$$x = 2$$

3b) $\log_2 8\sqrt{2}$

$$\log_2 8\sqrt{2} = x$$

$$2^x = 8\sqrt{2}$$

$$2^x = (2^3)(2^{1/2})$$

$$2^x = 2^{7/2}$$

$$x = \frac{7}{2}$$

$$3c) 2^{\log_2 7}$$

$$2^{\log_2 7} = x$$

$$\log_2 x = \log_2 7$$

$$x = 7$$

Ex 4) Solve each equation:

$$4a) \log_4 x = 3$$

$$4^3 = x$$

$$x = 64$$

$$4b) \log_x 81 = 4$$

$$x^4 = 81$$

$$\sqrt[4]{x^4} = \sqrt[4]{81}$$

$$x = 3$$