Law 1) For any positive numbers x and y, where a is any positive integer where $a \neq 1$

$$\log_a(x \cdot y) = \log_a x + \log_a y$$

Ex 1) Express as a sum of logarithms. Simplify, if possible.

$$\log_3(9 \cdot 27) \rightarrow \log_3 9 + \log_3 27 \rightarrow 2+3 \rightarrow 5$$

Ex 2) Express as a single logarithm.

$$\log_5 20 + \log_5 3 \rightarrow \log_5(20 \cdot 3) \rightarrow \log_5 60$$

Law 2) For any positive number x, any number p, and any logarithm base a,

$$\log_a x^p = p \cdot \log_a x$$

Express the following as a product:

Ex 3 $\log_b 9^{-5} = -5 \cdot \log_b 9$

EX 4 $\log_a \sqrt[4]{5} = \log_a 5^{\frac{1}{4}} = \frac{1}{4} \cdot \log_a 5$

Law 3) For any positive numbers x, y and any logarithm base a,

$$\log_a \frac{x}{y} = \log_a x - \log_a y$$

EX $4\frac{1}{2}$) Rewrite $\log_3 \frac{19}{5}$ as the difference of two logs.

$$\log_3 \frac{19}{5} = \log_3 19 - \log_3 5$$

Ex 5) Express in terms of logarithms of x, y and z.

$$\log_a \sqrt[4]{\frac{xy}{z^3}} \rightarrow \log_a \left(\frac{xy}{z^3}\right)^{\frac{1}{4}}$$

$$\frac{1}{4} \cdot \log_a \left(\frac{xy}{z^3}\right) \rightarrow \frac{1}{4}\left[\log_a xy - \log_a z^3\right]$$

$$\frac{1}{4}\left[\log_a x + \log_a y - 3 \cdot \log_a z\right]$$

$$\frac{1}{4}\log_a x + \frac{1}{4}\log_a y - \frac{3}{4}\log_a z$$

Ex 6) Express as a single logarithm:

$$\frac{1}{2}\log_a x - 7\log_a y + \log_a z$$

$$\log_a x^{\frac{1}{2}} - \log_a y^7 + \log_a z$$

$$\log_a \sqrt{x} - \log_a y^7 + \log_a z$$

$$\log_a\left(\frac{\sqrt{x}}{y^7}\right) + \log_a z$$

$$\log_a\left(\frac{\sqrt{x}\,z}{y^7}\right)$$

EX7 Given that $\log_a 2 \approx 0.301$, $\log_a 3 \approx 0.477$

7a) $\log_a 6 = \log_a(2 \cdot 3)$

$= \log_a 2 + \log_a 3$

$= 0.301 + 0.477$

$= 0.778$

7b) $\log_a \sqrt{3} = \log_a 3^{\frac{1}{2}}$

$= \frac{1}{2} \cdot \log_a 3$

$= \frac{1}{2}(0.477)$

≈ 0.2385

7c) $\log_a \frac{2}{3} = \log_a 2 - \log_a 3$

$\approx 0.301 - 0.477$

≈ -0.176

7d) $\log_a 5 \rightarrow$ can't do

7e) $\frac{\log_a 2}{\log_a 3} = \frac{0.301}{0.477}$

≈ 0.63