10-5 Laws (Properties) of Logarithms (Day 2)   Page 473

Law 1) For any positive numbers x and y, where a is any positive integer where a ≠ 1

$$\log_a (x \cdot y) = \log_a x + \log_a y$$

Law 2) For any positive number x, any number p, and any logarithm base a,

$$\log_a x^p = p \cdot \log_a x$$

Law 3) For any positive numbers x, y and any logarithm base a,

$$\log_a \frac{x}{y} = \log_a x - \log_a y$$

If $\log_a 5 \approx 0.67$ and $\log_a 7 \approx 0.81$, find $\log_a 25$

Strategy for solving this type of problem:

1) If you are given the base, use that information to find any known values. For example:

$$\log_3 1 = 0, \quad \log_3 3 = 1, \quad \log_3 9 = 2, \quad \text{etc.}$$

2) Determine if the number they are asking you to take the log of can be calculated by **multiplying** or **dividing** any of the numbers given, or by **raising** any of the numbers (or bases) given **to a power**. For example:

$$\log_a 175 = \log_a \left(5^2 \cdot 7 \right)$$

If $\log_a 5 \approx 0.67$ and $\log_a 7 \approx 0.81$, find the following:

1) $\log_a 25 = \log_a 5^2 = 2\log_a 5 \approx 2(0.67)$

$$\approx 1.34$$

2) $\log_a 35 = \log_a(5 \cdot 7) = \log_a 5 + \log_a 7$

$$\approx 0.67 + 0.81 \approx 1.48$$

3) $\log_a \frac{7}{5} = \log_a 7 - \log_a 5 \approx 0.81 - 0.67$

$$\approx 0.14$$

4) $\log_a 175 = \log_a(5^2 \cdot 7) = \log_a 5^2 + \log_a 7$

$$= 2\log_a 5 + \log_a 7$$
$$\approx 2(0.67) + 0.81$$
$$\approx 1.34 + 0.81$$
$$\approx 2.15$$

5) $\log_a 1 = 0$

6) $\log_a \sqrt{35} = \log_a 35^{\frac{1}{2}} = \frac{1}{2} \log_a 35$

$= \frac{1}{2} \log_a (5 \cdot 7) = \frac{1}{2} (\log_a 5 + \log_a 7)$

$\approx \frac{1}{2} (0.67 + 0.81)$

$\approx \frac{1}{2} (1.48)$

$\approx \boxed{0.74}$

7) $\log_a 12$ **_Can't be simplified unless we know the base. 12=5+7 which doesn't involve multiplication, division or exponents._**

8) $\log_a \frac{49}{25} = \log_a 49 - \log_a 25$

$= \log_a 7^2 - \log_a 5^2$

$= 2 \log_a 7 - 2 \log_a 5$

$\approx 2(0.81) - 2(0.67)$

$\approx 1.62 - 1.34$

$\approx \boxed{0.28}$