10-7 Exponential Growth and Decay P. 483

Formula for Exponential Growth:

$$N = ne^{kt}$$

k = constant of growth

t = time

e is the irrational number from the lesson 10-8

N: final amount (larger for growth)

n: initial amount (smaller)

Ex 1) For a certain strain of bacteria, k = 0.775 when t is measured in hours. How long will 2 bacteria take to increase to 1000 bacteria?

$$N = ne^{kt}$$

$$\frac{1000}{2} = \frac{2e^{0.775t}}{2}$$

$$500 = e^{0.775t}$$

(next, take the natural log of both sides of the equation.)

$$\ln 500 = \ln e^{0.775t}$$

$$6.2146 = 0.775t(\underbrace{\ln e}_{1})$$

$$\frac{0.775t}{0.775} = \frac{6.2146}{0.775}$$

$$t = 8.0188 \text{ hours}$$

$$t \approx 8 \text{ hours}$$

Formula for Decay:

$$n = N e^{kt}$$

constant of decay (should be a negative number)

t = time

e from lesson 10-8

final amount (smaller for decay)

initial amount (larger)

Ex 2) In 10 years, the mass of a 200 gram sample is reduced to 100 grams. This period is called the half-life of the sample (since half of the original amount remains). Find the constant k for this element.

$$n = Ne^{kt}$$

$$\frac{100}{200} = \frac{200e^{10k}}{200}$$

$$0.5 = e^{10k}$$ (next, take the natural log of both sides of the eq.)

$$\ln 0.5 = \ln e^{10k}$$

$$-0.6931 = 10k(\ln e)$$

$$\frac{10k}{10} = \frac{-0.6931}{10}$$

$$k = -0.06931$$

Ex 3) An isotope of the synthetic element Californium (no joke) has a half life of about 45 minutes. How long would it take for a given sample to decay and lose 85% of its original mass?

(Strategy: This problem requires 2 steps to solve.
First, find the constant of decay k, since this was not given. Let's start with 2 grams as our initial amount and 1 gram as our final amount (half our initial amount) and solve for k.
Second, Let's use 100 grams as our initial amount since we are dealing with percentages, and our final amount will be 15 grams (we lost 85% or 85 grams).

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$$n = Ne^{kt}$$

$$\frac{1}{2} = \frac{2e^{k45}}{2}$$

$$0.5 = e^{45k}$$

$$\ln 0.5 = \ln e^{45k}$$

$$-0.6931 = 45k(\ln e) \quad \ln e = 1$$

$$\frac{45k}{45} = \frac{-0.6931}{45}$$

$$\boxed{k = -0.0154}$$

II $n = Ne^{kt}$

$$15 = 100e^{-0.0154t}$$

$$\frac{15}{100} = \frac{100e^{-0.0154t}}{100}$$

$$0.15 = e^{-0.0154t}$$

$$\ln 0.15 = \ln e^{-0.0154t}$$

$$-1.8971 = -0.0154t\,(\ln e)$$

$$\frac{-1.8971}{-0.0154} = \frac{-0.0154t}{-0.0154}$$

$$t \doteq 123.1883$$

$$t \approx 123 \text{ minutes}$$

Compount Interest Formula:

If an initial amount **P** (called the principal) is invested at an annual interest rate ***r*** compounded ***n*** times a year, then in ***t*** years the interest will grow to a final amount **A**.

one

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

interest rate

time in years

Final Amount (Principal plus Interest)

initial amount

number of times the interest is compounded per year

Ex 4) How long will it take an investment of $1000 to triple in value if it is invested at an annual rate of 12% compounded quarterly (4 times a year).

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$\frac{3000}{1000} = \frac{1000\left(1 + \frac{0.12}{4}\right)^{4t}}{1000}$$

$$3 = (1 + 0.03)^{4t}$$

$$3 = 1.03^{4t}$$

$$\log 3 = \log 1.03^{4t}$$

$$0.4771 = 4t(\log 1.03)$$

$$0.4771 = 4t(0.0128)$$

$$\frac{0.4771}{0.0512} = \frac{0.0512t}{0.0512}$$

$$t \doteq 9.3184$$

$$t \approx 9.3 \text{ years}$$