Formula for Exponential Growth:

\[ N = n e^{k t} \]

- \( n \): Initial amount (smaller)
- \( e^{k t} \): The growth factor, where:
  - \( k \): Constant of growth
  - \( t \): Time
- \( e \): The irrational number from the lesson 10-8
Ex 1) For a certain strain of bacteria, \( k = 0.775 \) when \( t \) is measured in hours. How long will 2 bacteria take to increase to 1000 bacteria?

\[
N = n e^{kt}
\]

\[
1000 = 2 e^{0.775t}
\]

\[
\frac{1000}{2} = e^{0.775t}
\]

\[
500 = e^{0.775t}
\]

\[
\ln 500 = \ln e^{0.775t}
\]

\[
6.2146 = 0.775t \quad (\ln e)
\]

\[
0.775t = 6.2146
\]

\[
0.775 \quad 0.775
\]

\[
t = 8.0188 \text{ hours}
\]
Formula for Decay:

\[ n = N e^{-kt} \]

- \( n \): final amount (smaller for decay)
- \( N \): initial amount (larger)
- \( e \): from lesson 10-8
- \( k \): constant of decay (should be a negative number)
- \( t \): time
Ex 2) In 10 years, the mass of a 200 gram sample is reduced to 100 grams. This period is called the half-life of the sample (since half of the original amount remains). Find the constant $k$ for this element.

\[ n = Ne^{kt} \]

\[ \frac{100}{200} = e^{10k} \]

\[ 0.5 = e^{10k} \]

(next, take the natural log of both sides of the eq.)

\[ \ln 0.5 = \ln e^{10k} \]

\[ \ln 0.5 = 10k \ln e \]

\[ -0.6931 = 10k \ln e \]

\[ 10k = -0.6931 \]

\[ k = \frac{-0.6931}{10} \]

\[ k = -0.06931 \]
Ex 3) An isotope of the synthetic element Californium (no joke) has a half life of about 45 minutes. How long would it take for a given sample to decay and lose 85\% of its original mass?

(Strategy: This problem requires 2 steps to solve. First, find the constant of decay \( k \), since this was not given. Let's start with 2 grams as our initial amount and 1 gram as our final amount (half our initial amount) and solve for \( k \). Second, let's use 100 grams as our initial amount since we are dealing with percentages, and our final amount will be 15 grams (we lost 85\% or 85 grams).

\[
\frac{n}{N} = e^{kt}
\]

\[
\frac{1}{2} = 2 \cdot e^{k \cdot 45}
\]

\[
0.5 = e^{-45k}
\]

\[
\ln 0.5 = \ln e^{-45k}
\]

\[
-0.6931 = -45k \cdot (\ln 1/e)
\]

\[
45k = \frac{-0.6931}{-0.6931}
\]

\[
k = \frac{45}{-0.0154}
\]
(II) \[ n = N e^{kt} \]
\[ \frac{15}{100} = e^{-0.0154t} \]
\[ 0.15 = e^{-0.0154t} \]
\[ \ln 0.15 = \ln e^{-0.0154t} \]
\[ -1.8971 = -0.0154t \times (\ln 1) \]
\[ -0.0154 \quad -0.0154 \]

\[ t = 123.1883 \]

\[ t \approx 123 \text{ minutes} \]
Compount Interest Formula:
If an initial amount $P$ (called the principal) is invested at an annual interest rate $r$ compounded $n$ times a year, then in $t$ years the interest will grow to a final amount $A$.

$$A = P \left(1 + \frac{r}{n}\right)^{nt}$$

- $A$: Final amount
- $P$: Initial amount
- $r$: Interest rate
- $n$: Number of times the interest is compounded per year
- $t$: Time in years
Ex 4) How long will it take an investment of $1000 to triple in value if it is invested at an annual rate of 12% compounded quarterly (4 times a year).

\[ A = P \left(1 + \frac{r}{n}\right)^{nt} \]

\[ 3000 = 1000 \left(1 + \frac{0.12}{4}\right)^{4t} \]

\[
\frac{3000}{1000} = \left(1 + 0.03\right)^{4t} \\
3 = (1 + 0.03)^{4t} \\
3 = 1.03^{4t} \\
\log 3 = \log 1.03^{4t} \\
0.4771 = 4t(\log 1.03) \\
0.4771 = 4t(0.0128) \\
0.4771 = 0.0512t \\
\]

\[
t = 9.3184 \\
t \approx 9.3 \text{ years}
\]