10-7 Exponential Growth and Decay   P. 483

Alg. 2 Standard 12.0 Students know the laws of fractional exponents, understand exponential functions, and use these functions in problems involving exponential growth and decay.

Objective: To use exponential and logarithmic functions to solve growth and decay problems.

Doubling-Time Growth Formula:

If a population of size ***n*** doubles every ***d*** years (or hours, or days, or any other unit of time), then the number ***N*** in the population at time ***t*** is given using the following formula:

$$N = \underset{N_0}{n} \bullet 2^{\frac{t}{d}}$$

Ex 1) A certain bacteria population doubles in size every 12 hours. If we start with 2 bacteria, how many bacteria will we have after 2 days (48 hours)?

$$N = n \bullet 2^{\frac{t}{d}}$$

$$N = 2 \cdot 2^{\frac{48}{12}}$$

$$N = 2 \cdot 2^4$$

$$N = 2^5$$

$$N = 32$$

Ex 2) A certain bacteria population doubles in size every 12 hours. By how much will it grow in 2 days (48 hours)?

$$N = n \bullet 2^{\frac{t}{d}}$$

$$N = n \cdot 2^{\frac{48}{12}}$$

$$N = n \cdot 2^4$$

$$N = 16n$$

The population grows by a factor of 16 in 2 days.

Half-Live Decay Formula

If an amount ***N*** has a half-life ***h***, then the amount remaining at time ***t*** is represented by the formula:

$$n = N\left(\frac{1}{2}\right)^{\frac{t}{h}}$$

N N_0

Ex 3) The half-life of carbon-14 is 5730 years. How much of a 10.0 mg sample will remain after 4500 years?

$$n = N\left(\frac{1}{2}\right)^{\frac{t}{h}}$$

$$n = 10.0\left(\frac{1}{2}\right)^{\frac{4500}{5730}}$$

$$n = 10.0\left(\frac{1}{2}\right)^{0.7853}$$

$$n = 10.0(0.5802)$$

$$n = 5.802 \text{ mg}$$

Solution 2

$$\log N = \log 10.0 + \frac{4500}{5730}\log 0.5$$

$$\log N = 1 + (0.7853)(-0.3010)$$

$$\log N = 0.7636$$

$$N = 5.80$$