10-7 Exponential Growth and Decay   P. 483

Alg. 2 Standard 12.0 Students know the laws of fractional exponents, understand exponential functions, and use these functions in problems involving exponential growth and decay.

Objective: To use exponential and logarithmic functions to solve growth and decay problems.

Doubling-Time Growth Formula:

If a population of size ***n*** doubles every ***d*** years (or hours, or days, or any other unit of time), then the number ***N*** in the population at time ***t*** is given using the following formula:

$$N = n \bullet 2^{\frac{t}{d}}$$

N_0

Ex 1) A certain bacteria population doubles in size every 12 hours. If we start with 2 bacteria, how many bacteria will we have after 2 days (48 hours)?

$$N = n \bullet 2^{\frac{t}{d}}$$

$$N = 2 \cdot 2^{\frac{48}{12}}$$

$$N = 2 \cdot 2^4$$

$$N = 2^5$$

$$N = 32$$

Ex 2) A certain bacteria population doubles in size every 12 hours. By how much will it grow in 2 days (48 hours)?

$$N = n \bullet 2^{\frac{t}{d}}$$

$$N = n \cdot 2^{\frac{48}{12}}$$

$$N = n \cdot 2^4$$

$$N = 16n$$

The population grows by a factor of 16 in 2 days.

Half-Live Decay Formula

If an amount $\boldsymbol{N}$ has a half-life $\boldsymbol{h}$, then the amount remaining at time $\boldsymbol{t}$ is represented by the formula:

$$n = N\left(\frac{1}{2}\right)^{\frac{t}{h}}$$

N N_o

Ex 3) The half-life of carbon-14 is 5730 years. How much of a 10.0 mg sample will remain after 4500 years?

$$n = N\left(\frac{1}{2}\right)^{\frac{t}{h}}$$

$$n = 10.0\left(\frac{1}{2}\right)^{\frac{4500}{5730}}$$

$$n = 10.0\left(\frac{1}{2}\right)^{0.7853}$$

$$n = 10.0(0.5802)$$

$$n = 5.802 \text{ mg}$$

Solution 2

$$\log N = \log 10.0 + \frac{4500}{5730}\log 0.5$$

$$\log N = 1 + (0.7853)(-0.3010)$$

$$\log N = 0.7636$$

$$N = 5.80$$

Recall the formula for **Simple Interest** from previous math classes:

$I = P \cdot R \cdot T$ and $A = P + I$

Where ***I*** represents ***interest***, ***P*** represents ***principal***, ***T*** is ***time*** (usually in years), and ***A*** is the total ***amount***.

Ex 4) If \$1000 is invested at 3% for a period of 5 years, how much interest was earned? What is the total amount?

$I = P \cdot R \cdot T$

$I = (1000)(0.03)(5)$

$I = \$150$

$A = P + I$

$A = 1000 + 150$

$A = \$1150$

Compound Interest

If an amount ***P*** (called the principal) is invested at an annual interest rate ***r*** (expressed as a decimal) compounded ***n*** times a year, then in ***t*** years the investment will grow to an amount ***A*** using this formula.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$I = A - P$$

Ex 5) How long will it take for an investment of \$1000 to triple in value if it is invested at an annual rate of 12% compounded quarterly?

Let P = 1000, A = 3000, r = 0.12, and n=4.

$$\frac{3000}{1000} = \frac{1000\left(1 + \frac{0.12}{4}\right)^{4t}}{1000}$$

$$3 = 1.03^{4t}$$

$$\log 3 = \log 1.03^{4t}$$

$$\frac{\log 3}{4 \log 1.03} = \frac{4t \log 1.03}{4 \log 1.03}$$

$$t = \frac{\log 3}{4 \log 1.03}$$

$$t = \frac{0.4771}{4(0.0128)}$$

$$t = \frac{0.4771}{0.0512}$$

$t \approx 9.3$ years