Alg. 2 Standard 12.0 Students know the laws of fractional exponents, understand exponential functions, and use these functions in problems involving exponential growth and decay.

Objective: To use exponential and logarithmic functions to solve growth and decay problems.

Doubling-Time Growth Formula:
If a population of size \( n \) doubles every \( d \) years (or hours, or days, or any other unit of time), then the number \( N \) in the population at time \( t \) is given using the following formula:

\[
N = n \cdot 2^{\frac{t}{d}}
\]
Ex 1) A certain bacteria population doubles in size every 12 hours. If we start with 2 bacteria, how many bacteria will we have after 2 days (48 hours)?

\[ N = n \cdot 2^{\frac{t}{12}} \]

\[ N = 2 \cdot 2^{\frac{48}{12}} \]

\[ N = 2 \cdot 2^4 \]

\[ N = 2^5 \]

\[ N = 32 \]

Ex 2) A certain bacteria population doubles in size every 12 hours. By how much will it grow in 2 days (48 hours)?

\[ N = n \cdot 2^{\frac{t}{12}} \]

\[ N = n \cdot 2^{\frac{48}{12}} \]

\[ N = n \cdot 2^4 \]

\[ N = 16n \]

The population grows by a factor of 16 in 2 days.
Half-Life Decay Formula
If an amount $N$ has a half-life $h$, then the amount remaining at time $t$ is represented by the formula:

$$n = N \left( \frac{1}{2} \right)^{t/h}$$

$N$ $N_0$

Ex 3) The half-life of carbon-14 is 5730 years. How much of a 10.0 mg sample will remain after 4500 years?

$$n = 10.0 \left( \frac{1}{2} \right)^{4500/5730}$$

$$n = 10.0 \left( \frac{1}{2} \right)^{0.7853}$$

$$n = 10.0 \left( 0.5802 \right)$$

$$n = 5.802 \text{ mg}$$
Solution 2

\[ \log N = \log 10.0 + \frac{4500}{5730} \log 0.5 \]

\[ \log N = 1 + (0.7853)(-0.3010) \]

\[ \log N = 0.7636 \]

\[ N = 5.80 \]

Recall the formula for **Simple Interest** from previous math classes:

\[ I = P \cdot R \cdot T \quad \text{and} \quad A = P + I \]

Where \( I \) represents **interest**, \( P \) represents **principal**, \( T \) is **time** (usually in years), and \( A \) is the total **amount**.
Ex 4) If $1000 is invested at 3% for a period of 5 years, how much interest was earned? What is the total amount?

\[ I = P \cdot R \cdot T \]

\[ I = (1000)(0.03)(5) \]

\[ I = \$150 \]

\[ A = P + I \]

\[ A = 1000 + 150 \]

\[ A = \$1150 \]

**Compound Interest**
If an amount \( P \) (called the principal) is invested at an annual interest rate \( r \) (expressed as a decimal) compounded \( n \) times a year, then in \( t \) years the investment will grow to an amount \( A \) using this formula.

\[ A = P \left(1 + \frac{r}{n}\right)^{nt} \]

\[ I = A - P \]
Ex 5) How long will it take for an investment of $1000 to triple in value if it is invested at an annual rate of 12% compounded quarterly?

Let $P = 1000$, $A = 3000$, $r = 0.12$, and $n=4$.

$$3000 = \frac{1000}{1000} \left(1 + \frac{0.12}{4}\right)^{4t}$$

$$\frac{3000}{1000} = \left(1.03\right)^{4t}$$

$$3 = \left(1.03\right)^{4t}$$

$$\log 3 = \log \left(1.03\right)^{4t}$$

$$\log 3 = 4t \log 1.03$$

$$4 \log 1.03$$

$$4 \log 1.03$$

$$t = \frac{\log 3}{4 \log 1.03}$$
\[ t = \frac{0.4771}{\frac{1}{4}(0.0128)} \]

\[ t = \frac{0.4771}{0.0512} \]

\[ t \approx 9.3 \text{ years} \]