10-8 Natural Logarithms Page 489

There is an important irrational number called ***e***, which occurs in advanced mathematics, statistics, probability, and many other situations involving growth.

$e \approx 2.718281828459...$

Logarithms base ***e*** are called natural logarithms and are written as: **ln** x.

In the past we used to look up natural logarithms in a table much the same way we used a table to find logs base 10 (or common logs). Below you will see a portion of a table of natural logs.

Natural Logarithms (ln x)

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
1.0	0.0000	0.0100	0.0198	0.0296	0.0392	0.0488	0.0583	0.0677	0.0770	0.0862
1.1	0.0953	0.1044	0.1133	0.1222	0.1310	0.1398	0.1484	0.1570	0.1655	0.1740
1.2	0.1823	0.1906	0.1989	0.2070	0.2151	0.2231	0.2311	0.2390	0.2469	0.2546
1.3	0.2624	0.2700	0.2776	0.2852	0.2927	0.3001	0.3075	0.3148	0.3221	0.3293
1.4	0.3365	0.3436	0.3507	0.3577	0.3646	0.3716	0.3784	0.3853	0.3920	0.3988

Often, the number we are looking for will not be found on the table and we will have to perform calculations known as interpolation.

We will use our scientific calculators to find natural logs.

Ex 1) Rewrite the following natural logs in exponential form and the following exponents in natural logarithmic form.
Remember: natural logs are base *e*

1a) $\ln x = 5 \rightarrow e^5 = x$

1b) $e^x = 7 \rightarrow \ln 7 = x$

1c) $\ln 12 = x \rightarrow e^x = 12$

Ex2) Simplify:

2a) $\ln \frac{1}{e^2} \rightarrow \ln e^{-2} \rightarrow -2(\ln e)$

$-2(1) = -2$

2b) Write as a single logarithm:

$2\ln 5 + \ln 4 - 3$

$\ln 5^2 + \ln 4 - 3(\ln e)$ (Remember $\ln e = 1$)

$\ln 25 + \ln 4 - \ln e^3$

$\ln(25 \cdot 4) - \ln e^3$

$\ln 100 - \ln e^3$

$\ln\left(\frac{100}{e^3}\right)$

Let's use our calculators to calculate some natural logs and their inverses.

Ex3 $\ln 5.24 \doteq 1.6563$

Ex4 $\ln 0.001277 \doteq -6.6632$

Ex5 $\ln 1{,}234 \doteq 7.1180$

EX6 $e^1 \doteq 2.71828$

EX7 $e^{1.23} \doteq 3.4212$

EX8 Solve:

8a) $\ln_e x = 2$

$e^2 = x$

$x \approx 7.3891$

8b) $\ln \frac{1}{x} = 2$

$e^2 = \frac{1}{x}$

$x = \frac{1}{e^2}$

$x = e^{-2}$

$x \approx 0.1353$

EX 9 Solve $e^{2x} = 9$

Take the natural log of both sides.

$$\ln e^{2x} = \ln 9$$

$$2x(\ln e) = \ln 9$$

$$2x \approx 2.1972$$

$$x \approx 1.0986$$