The ***n***th term of an arithmetic sequence is given by:

$$t_n = t_1 + (n-1)d$$

Ex 1) Find the formula for the ***n***th term of the sequence: 5, 8, 11, 14, . . .

$t_n = t_1 + (n-1)d$

The first term, t_1 is: 5

The common difference, d is: 3 (8-5, 11-8, 14-11)

$t_n = 5 + (n-1)3$

Ex 2) Find the 15th term of the sequence: 4, 9, 14, 19, . . .
in other words find: t_{15}

$$t_n = t_1 + (n-1)d$$

The common difference, d is: 5

The first term, t_1 is: 4

$t_{15} = 4 + (15 - 1)\,5$

$t_{15} = 4 + (14)\,5$

$t_{15} = 4 + 70$

$t_{15} = 74$

Ex 3) In the arithmetic sequence 4, 7, 10, 13, . . . , which term has a value of 301?

$$t_n = t_1 + (n-1)d$$

$d = 3$

$t_1 = 4$

$301 = 4 + (n-1)\,3$

$301 = 4 + 3n - 3$

$301 = 3n + 1$

$300 = 3n$

$100 = n$

or

$t_{100} = 301$

Ex 4) The 3rd term of an arithmetic sequence is 8 and the 16th term is 47. Find the 25th term of the sequence.

$$t_n = t_1 + (n-1)d$$

$t_3 = 8$

$t_{16} = 47$

Strategy: Since the above information cannot be used in one equation to solve the problem, we will need to set up 2 equations with 2 variables to find the formula for the general term of the sequence. Once we have the formula for the general term, we can then find the 25th term.

$8 = t_1 + (3-1)d$

$47 = t_1 + (16-1)d$

Simplify each equation:

$8 = t_1 + 2d$

$47 = t_1 + 15d$

Solve the system of simultaneous equations using any of the methods that we have previously learned (graphing, substitution, linear combination).
Let's use linear combination, which we also called the elimination method.

$$t_1 + 15d = 47$$
$$+(-t_1 + -2d = -8)$$

$$13d = 39$$

$$\boxed{d = 3}$$

Next, let's find the first term by substituting 3 in for d:

$$t_1 + 2(3) = 8$$

$$t_1 + 6 = 8$$

$$\boxed{t_1 = 2}$$

The general formula for the sequence is:

$$t_n = 2 + (n-1)3$$

We can now find the 25th term of the sequence.

$$t_{25} = 2 + (25-1)3$$

$$t_{25} = 2 + (24)3$$

$$t_{25} = 2 + 72$$

$$\boxed{t_{25} = 74}$$

A simple **arithmetic mean** between 2 numbers is the average of the two numbers $(a + b) / 2$.

Ex 5a) Find the arithmetic mean between 7 and 19.

$$\frac{7+19}{2} \rightarrow \frac{26}{2} \rightarrow \boxed{13}$$

Ex 5b) Insert three arithmetic means between 8 and 16.

8, __, __, __, 16

Strategy: 8 is the first term of the sequence, while 16 is the fifth term of the sequence. Let's use todays formula for arithmetic sequences:

$$t_n = t_1 + (n-1)d$$

$16 = 8 + (5-1)d$

$16 = 8 + 4d$

$8 = 4d$

$\boxed{d = 2}$

8, 10, 12, 14, 16

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