

Objective: To find a formula for the n th term of a geometric sequence and to find specified terms of the geometric sequences.

Definition: A sequence in which a constant r can be multiplied by each term to get the next is called a **geometric sequence**. The constant r is called the **common ratio**.

In a geometric sequence, the n th term is given by the formula:

$$t_n = t_1 \bullet r^{n-1}$$

Ex 1) Find a formula for the n th term of the sequence:
3, -12, 48, -192, ...

$$t_n = t_1 \bullet r^{n-1}$$

$$t_1 = 3$$

$$r = -4 \quad (-12 \div 3, 48 \div -12, -192 \div 48)$$

$$t_n = 3(-4)^{n-1}$$

Ex 2) Find the 11th term of the geometric sequence:

64, -32, 16, -8, ...

$$t_n = t_1 \bullet r^{n-1}$$

$$t_1 = 64$$

$$r = -\frac{1}{2} \quad (-32 \div 64, 16 \div -32, -8 \div 16)$$

$$t_{11} = 64 \left(-\frac{1}{2}\right)^{(11-1)}$$

$$t_{11} = 64 \left(-\frac{1}{2}\right)^{10}$$

$$t_{11} = 64 \left(\frac{1}{2^{10}}\right)$$

$$t_{11} = 2^6 \cdot 2^{-10}$$

$$t_{11} = 2^{-4} \text{ or } \frac{1}{16}$$

Ex 3) The second term of a geometric sequence is 24, the fifth term is 3, find the seventh term of the sequence.

$$t_n = t_1 \cdot r^{n-1}$$

$$t_2 = 24, t_5 = 3$$

$$\textcircled{\text{I}} \begin{aligned} 24 &= t_1 \cdot r^{2-1} \\ 3 &= t_1 \cdot r^{5-1} \end{aligned}$$

$$\begin{aligned} 24 &= t_1 r \\ 3 &= t_1 r^4 \end{aligned}$$

$$t_1 = \frac{24}{r} \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{substitution}$$
$$3 = \left(\frac{24}{r} \right) r^4$$

$$3 = 24r^3$$

$$\frac{1}{8} = r^3$$

$$\sqrt[3]{\frac{1}{8}} = \sqrt[3]{r^3}$$

$$r = \frac{1}{2}$$

II

$$24 = t_1 \left(\frac{1}{2}\right)$$

$$48 = t_1$$

$$t_n = 48 \cdot \left(\frac{1}{2}\right)^{n-1}$$

$$t_7 = 48 \cdot \left(\frac{1}{2}\right)^{7-1}$$

$$t_7 = 48 \cdot \left(\frac{1}{2}\right)^6$$

$$t_7 = 48 \cdot \frac{1}{64}$$

$$t_7 = \frac{3}{4}$$

The geometric mean of two numbers a and b is the square root of their product, or: \sqrt{ab}

Ex 4a) Find the geometric mean of 4 and 9.

$$\sqrt{4 \cdot 9} \rightarrow \sqrt{36} \rightarrow \textcircled{6}$$

Ex 4b) Insert two geometric means between 3 and 24.

3, __, __, 24

$$t_1 = 3 \quad t_4 = 24$$

$$t_n = t_1 \cdot r^{n-1}$$

$$24 = 3 \cdot r^{4-1}$$

$$24 = 3r^3$$

$$8 = r^3$$

$$\sqrt[3]{8} = \sqrt[3]{r^3}$$

$$\boxed{r = 2}$$

3, 6, 12, 24

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