11-3 Geometric Sequences Page 510

Alg. 2 Standard 22.0 **Students find the general term and the sums** of arithmetic series and of both **finite** and infinite **geometric series.**

Objective: To find a formula for the ***n***th term of a geometric sequence and to find specified terms of the **geometric sequences.**

Definition: A sequence in which a constant ***r*** can be multiplied by each term to get the next is called a **geometric sequence**. The constant ***r*** is called the **common ratio.**

In a geometric sequence, the ***n***th term is given by the formula:

$$t_n = t_1 \bullet r^{n-1}$$

t_n is the n^{th} term, t_1 is the first term, n is the number of terms, r is the common ratio.

Ex 1) Find a formula for the ***n***th term of the sequence: 3, -12, 48, -192, . . .

$$t_n = t_1 \bullet r^{n-1}$$

$t_1 = 3$

$r = -4$ $(-12 \div 3,\ 48 \div -12,\ -192 \div 48)$

$t_n = 3(-4)^{n-1}$

Ex 2) Find the 11th term of the geometric sequence:
64, -32, 16, -8, . . .

$$t_n = t_1 \bullet r^{n-1}$$

$t_1 = 64$

$r = -\frac{1}{2}$ $(-32 \div 64,\ 16 \div -32,\ -8 \div 16)$

$t_{11} = 64\left(-\frac{1}{2}\right)^{(11-1)}$

$t_{11} = 64\left(-\frac{1}{2}\right)^{10}$

$t_{11} = 64\left(\frac{1}{2^{10}}\right)$

$t_{11} = 2^6 \cdot 2^{-10}$

$t_{11} = 2^{-4}$ or $\frac{1}{16}$

Ex 3) The second term of a geometric sequence is 24, the fifth term is 3, find the seventh term of the sequence.

$$t_n = t_1 \bullet r^{n-1}$$

$t_2 = 24$, $t_5 = 3$

(I) $24 = t_1 \cdot r^{2-1}$

$3 = t_1 \cdot r^{5-1}$

$24 = t_1 \cdot r$

$3 = t_1 \cdot r^4$

Div. 2 by 5

$t_1 = \frac{24}{r}$ substitution

$3 = \left(\frac{24}{r}\right) r^4$

$3 = 24r^3$

$\frac{1}{8} = r^3$

$\sqrt[3]{\frac{1}{8}} = \sqrt[3]{r^3}$

$r = \frac{1}{2}$

II $24 = t_1\left(\frac{1}{2}\right)$

$48 = t_1$

$t_n = 48 \cdot \left(\frac{1}{2}\right)^{n-1}$

III $t_7 = 48 \cdot \left(\frac{1}{2}\right)^{7-1}$

$t_7 = 48 \cdot \left(\frac{1}{2}\right)^{6}$

$t_7 = 48 \cdot \frac{1}{64}$

$t_7 = \frac{3}{4}$

The geometric mean of two numbers ***a*** and ***b*** is the square root of their product, or: $\sqrt{ab}$

Ex 4a) Find the geometric mean of 4 and 9.

$\sqrt{4 \cdot 9} \rightarrow \sqrt{36} \rightarrow 6$

Ex 4b) Insert two geometric means between 3 and 24.

3, ___, ___, 24

$t_1 = 3 \quad t_4 = 24$

$$t_n = t_1 \bullet r^{n-1}$$

$$24 = 3 \cdot r^{4-1}$$

$$24 = 3r^3$$

$$8 = r^3$$

$$\sqrt[3]{8} = \sqrt[3]{r^3}$$

$$\boxed{r = 2}$$

3, 6, 12, 24

2 2 2

Ex 5) George has taken a job with a starting salary of $20,000. Find his salary during his 4th year on the job if he receives annual raises of: a) $1,100 and b) 5%.

a) an annual raise of $1,100 forms an arithmetic sequence.

$t_1 = 20{,}000$ $d = 1{,}100$ $t_n = t_1 + (n-1)d$

$t_4 = 20{,}000 + (4-1)\,1{,}100$

$= 20{,}000 + 3{,}300 = \$23{,}300$

b) an annual raise of 5% forms a geometric sequence.

$t_1 = 20{,}000$ $r = 1.05$ (not 0.05; $S + .05S = 1.05S$)

S = salary

$t_n = t_1 \cdot r^{n-1}$

$t_4 = 20{,}000(1.05)^{4-1}$

$= 20{,}000(1.05)^3$

$= 20{,}000(1.1576)$

$= \$23{,}152$

Should George ask for the annual raise of $1100 or 5%?