An **arithmetic series** is the sum of the terms of an arithmetic sequence, while a **geometric series** is the sum of the terms of a geometric sequence.

A series can be written in a shortened form using the Greek letter Σ (sigma), called the summation sign. For example, instead of writing $2 + 4 + 6 + 8 + \ldots + 100$, we can write:

\[
\sum_{n=1}^{50} 2n
\]

This series begins with the term for $n = 1$, and ends with the term for $n = 50$. This notation is read as "the sum of $2n$ for the values of $n$ from 1 to 50". The general term $2n$ is called the **summand**, while the letter $n$ is called the **index**.

\[
\sum_{n=1}^{50} 2n = 2 \cdot 1 + 2 \cdot 2 + 2 \cdot 3 + 2 \cdot 4 + \ldots + 2 \cdot 50
\]

The first and last values of the index are called the limits of summation. In this example, 1 is the lower limit and 50 is the upper limit. In an infinite series we use the symbol $\infty$ to represent the upper limit, indicating that the summation does not end.
Ex 1) Write the following series in expanded form:

\[
\sum_{j=1}^{20} (-1)^j (j + 2) = (-1)^1(1+2) + (-1)^2(2+2) + (-1)^3(3+2) + \cdots + (-1)^{20}(20+2)
\]

\[
= -3 + 4 + -5 + \cdots + 22
\]

Ex 2) Write the following series in expanded form:

\[
\sum_{k=1}^{\infty} \frac{1}{2^{(k-1)}} = \frac{1}{2^{1-1}} + \frac{1}{2^{2-1}} + \frac{1}{2^{3-1}} + \cdots
\]

\[
= \frac{1}{2^0} + \frac{1}{2^1} + \frac{1}{2^2} + \cdots
\]

\[
= 1 + \frac{1}{2} + \frac{1}{4} + \cdots
\]
Ex 3) Use sigma notation to write the following series:
10 + 15 + 20 + \ldots + 100.

Strategy: This series includes multiples of 5 from 10 to 100, let's factor out the 5s.

\[ 2.5 + 3.5 + 4.5 + \ldots + 20.5 = \sum_{n=2}^{20} 5n \]

or Solution 2:
common difference, \(d:5\)
The first term is: 10
The \(n^{th}\) term is:
\[ t_n = 10 + (n-1)5 \]
\[ = 10 + 5n - 5 \]
\[ = 5n + 5 \]
\[ = 5(n+1) \]

Find \(n\) if the last term is 100
\[ t_n = 5n + 5 \]
\[ 100 = 5n + 5 \]
\[ 95 = 5n \]
\[ n = 19 \]

\[ \sum_{k=1}^{19} 5(k + 1) \]
Ex 4) Use sigma notation to write the following series:

\[ \frac{5}{2} - \frac{5}{4} + \frac{5}{6} - \frac{5}{8} + \ldots \]

\[ \sum_{n=1}^{8} (-1)^{n+1} \left( \frac{5}{2n} \right) \]