An **arithmetic series** is the sum of the terms of an arithmetic sequence, while a **geometric series** is the sum of the terms of a geometric sequence.

A series can be written in a shortened form using the Greek letter Σ (sigma), called the summation sign.
For example, instead of writing 2 + 4 + 6 + 8 + ... + 100, we can write:

$$\sum_{n=1}^{50} 2n$$

This series begins with the term for $n = 1$, and ends with the term for $n = 50$. This notation is read as "the sum of $2n$ for the values of n from 1 to 50". The general term $2n$ is called the **summand,** while the letter n is called the **index.**

$$\sum_{n=1}^{50} 2n = 2 \cdot 1 + 2 \cdot 2 + 2 \cdot 3 + 2 \cdot 4 + \ldots + 2 \cdot 50$$

The first and last values of the index are called the limits of summation. In this example, 1 is the lower limit and 50 is the upper limit. In an infinite series we use the symbol ∞ to represent the upper limit, indicating that the summation does not end.

Ex 1) Write the following series in expanded form:

$$\sum_{j=1}^{20}(-1)^j(j+2) = (-1)^1(1+2) + (-1)^2(2+2) + (-1)^3(3+2) + \cdots + (-1)^{20}(20+2)$$

$$= -3 + 4 + -5 + \cdots + 22$$

Ex 2) Write the following series in expanded form:

$$\sum_{k=1}^{\infty}\frac{1}{2^{(k-1)}} = \frac{1}{2^{1-1}} + \frac{1}{2^{2-1}} + \frac{1}{2^{3-1}} + \cdots$$

$$= \frac{1}{2^0} + \frac{1}{2^1} + \frac{1}{2^2} + \cdots$$

$$= 1 + \frac{1}{2} + \frac{1}{4} + \cdots$$

Ex 3) Use sigma notation to write the following series:
10 + 15 + 20 + . . . + 100.

Strategy: This series includes multiples of 5 from 10 to 100, let's factor out the 5s.

$$2 \cdot 5 + 3 \cdot 5 + 4 \cdot 5 + \cdots + 20 \cdot 5 = \sum_{n=2}^{20} 5n$$

OR Solution 2:

common difference, d: 5

The first term is: 10

The n^{th} term is: $t_n = 10 + (n-1)5$

$$= 10 + 5n - 5$$

$$= 5n + 5$$

$$= 5(n+1)$$

Find n if the last term is 100

$t_n = 5n + 5$

$100 = 5n + 5$

$95 = 5n$

$n = 19$

$$\sum_{k=1}^{19} 5(k+1)$$

Ex 4) Use sigma notation to write the following series:

$$\frac{5}{2} - \frac{5}{4} + \frac{5}{6} - \frac{5}{8} + \cdots$$

$$\sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{5}{2n}\right)$$