The sum of the first ***n*** terms of an arithmetic series is:

$$S_n = \frac{n(t_1 + t_n)}{2}$$

This formula is useful when you know the first and last numbers is the series, and how many terms there are in the series.

Ex 1) Find the sum of the positive integers from 1 to 100.

$$S_n = \frac{n(t_1 + t_n)}{2}$$

$$S_{100} = \frac{100(1 + 100)}{2}$$

$$S_{100} = \frac{100(101)}{2}$$

$$S_{100} = \frac{10100}{2}$$

$$S_{100} = 5050$$

The sum of the first ***n*** terms of an arithmetic series can also be calculated using this formula:

$$S_n = \frac{n}{2}\left[2t_1 + (n-1)d\right]$$

This formula is useful when you know the first term and the common difference of the arithmetic series, but do not know the last term of the series.

Ex 2) Find the sum of the first 40 terms of the arithmetic series:
2 + 5 + 8 + 11 + . . .

$$S_n = \frac{n}{2}\left[2t_1 + (n-1)d\right] \qquad t_1 = 2,\ d = 3$$

$$S_{40} = \frac{40}{2}\left[2\cdot 2 + (40-1)3\right]$$

$$S_{40} = 20\left[4 + (39)3\right]$$

$$S_{40} = 20\left[4 + 117\right]$$

$$S_{40} = 20\left[121\right]$$

$$S_{40} = 2420$$

Ex 3) Evaluate:

$$\sum_{k=1}^{20}(5k+2)$$

$$S_n = \frac{n(t_1+t_n)}{2}$$

We will use this formula since we can easily calculate the first and last terms in this series.

$t_1 = 5(1)+2 \rightarrow 5+2 \rightarrow 7$

$t_{20} = 5(20)+2 \rightarrow 100+2 \rightarrow 102$

$n = 20$

$$S_{20} = \frac{20(7+102)}{2}$$

$$S_{20} = \frac{20(109)}{2}$$

$$S_{20} = \frac{2180}{2} = 1090$$

The sum of the first ***n*** terms of a geometric series with a common ratio ***r*** (where r $\neq 1$) is:

$$S_n = \frac{t_1(1-r^n)}{1-r}$$

Ex 4) Evaluate:

$$\sum_{n=1}^{10} 3(-2)^{n-1}$$

$$S_n = \frac{t_1\left(1-r^n\right)}{1-r}$$

$$t_1 = 3(-2)^{1-1} \rightarrow 3(-2)^0 \rightarrow 3 \cdot 1 \rightarrow 3$$

$$r = -2$$

$$S_{10} = \frac{3\left(1-(-2)^{10}\right)}{1-(-2)}$$

$$S_{10} = \frac{3(1-1024)}{1++2}$$

$$S_{10} = \frac{\cancel{3}(-1023)}{\cancel{3}}$$

$$S_{10} = -1023$$