11-6 Infinite Geometric Series   Page 531

If an infinite series approaches some limit as the number of terms ***n*** becomes very large, that limit is defined to be the sum of the series. If an infinite series has a sum it is said to converge or to be **convergent.**

Theorem: An infinite geometric series is **convergent** and has a sum ***S*** if and only if it's common ratio, ***r*** meets the following condition: $| r | < 1$

Consider the following geometric series:
$2 + 4 + 8 + 16 + 32 + 64 + \ldots$

As the number of terms, ***n***, gets larger, each term gets larger so that eventually for some large ***n*,** the term approaches infinity. The sum of this series also approaches infinity and is not convergent.

As we can see the common ratio $r = 2$ and $| 2 |$ is not less than 1.

Consider the following geometric series:

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\frac{1}{64}+\frac{1}{128}+\frac{1}{256}+\frac{1}{512}+\ldots+\frac{1}{2^n}$$

$$S_1=\frac{1}{2}$$

$$S_2=\frac{1}{2}+\frac{1}{4}=\frac{3}{4}$$

$$S_3=\frac{1}{2}+\frac{1}{4}+\frac{1}{8}=\frac{7}{8}$$

$$S_4=\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}=\frac{15}{16}$$

$$S_{10}=\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\ldots+\frac{1}{1024}=\frac{1023}{1024}$$

The more terms we add to our ***partial sums*** above, the closer the series "***approaches a limit***" of 1.

If our infinite series is **convergent** ($|r| < 1$) we can calculate its sum by the formula:

$$S = \frac{t_1}{1-r}$$

Do the following geometric series have a sum? If so, find the sum:

Ex 1) $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} + \ldots$

$r = -\frac{1}{2}$, $\left| -\frac{1}{2} \right| < 1$ is true

$$S = \frac{t_1}{1-r}$$

$$S = \frac{1}{1 - -\frac{1}{2}} = \frac{1}{1+\frac{1}{2}} = \frac{1}{\frac{3}{2}} = \frac{2}{3}$$

Ex 2) 1 + 5 + 25 + 125 + . . .

$r = 5$, $|5| < 1$ is False

This series does NOT have a sum.

Ex 3) 1 + (-1) + 1 + (-1) + . . .

$r = -1$, $|-1| < 1$ is False

This series does NOT have a sum.

Ex 4) $5+\frac{5}{2}+\frac{5}{4}+\frac{5}{8}+\ldots$

$r = \frac{1}{2}$, $|\frac{1}{2}| < 1$ is True

$$S=\frac{t_1}{1-r}$$

$$S = \frac{5}{1-\frac{1}{2}} = \frac{5}{\frac{1}{2}} = 5 \cdot 2 = 10$$