The Binomial Theorem

For any binomial (a + b) and any whole number ***n***, then $(a+b)^n$ =

$$= a^n + \frac{n}{1}a^{n-1}b + \frac{n(n-1)}{1\bullet 2}a^{n-2}b^2 + \frac{n(n-1)(n-2)}{1\bullet 2\bullet 3}a^{n-3}b^3 + \ldots + b^n$$

OR

$$= a^n + \frac{n!}{(n-1)!1!}a^{n-1}b + \frac{n!}{(n-2)!2!}a^{n-2}b^2 + \frac{n!}{(n-3)!3!}a^{n-3}b^3 + \ldots + b^n$$

OR

$$= \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \binom{n}{3}a^{n-3}b^3 + \ldots + \binom{n}{n}b^n$$

OR

$$= {}_nC_0\, a^n + {}_nC_1\, a^{n-1}b + {}_nC_2\, a^{n-2}b^2 + {}_nC_3\, a^{n-3}b^3 + \ldots + {}_nC_n\, b^n$$

Combinations:

$$\binom{n}{r} = {}_nC_r = \frac{n!}{(n-r)!r!}$$

Factorial:

$$n! = n\cdot(n-1)\cdot(n-2)\cdot\ldots 2\cdot 1$$

Look for the following buttons on your calculator:

$n!$ ${}_nC_r$

Ex 1) Find the first four terms of the binomial expansion:

$$(a+b)^{15}$$

$$\binom{15}{0}a^{15}+\binom{15}{1}a^{14}b+\binom{15}{2}a^{13}b^2+\binom{15}{3}a^{12}b^3$$

$$a^{15}+15a^{14}b+105a^{13}b^2+455a^{12}b^3$$

Ex 2) Find the first five terms of the binomial expansion:

$$(a-b)^{12}$$

$$\binom{12}{0}a^{12}+\binom{12}{1}a^{11}(-b)+\binom{12}{2}a^{10}(-b)^2+\binom{12}{3}a^9(-b)^3+\binom{12}{4}a^8(-b)^4$$

$$a^{12}-12a^{11}b+66a^{10}b^2-220a^9b^3+495a^8b^4$$

To find the ***r***th term of a binomial expansion raised to the ***n***th power, use the following formula:

$$\binom{n}{r-1}a^{(n-r+1)}b^{(r-1)}$$

Ex 3) Find the 7^{th} term of $(4x - y^2)^9$

$$\binom{n}{r-1} a^{(n-r+1)} b^{(r-1)}$$

$$\binom{9}{7-1} (4x)^{(9-7+1)} (-y^2)^{(7-1)}$$

$$\binom{9}{6} (4x)^3 (-y^2)^6$$

$$(84)(64x^3)(y^{12})$$

$$5376\, x^3 y^{12}$$