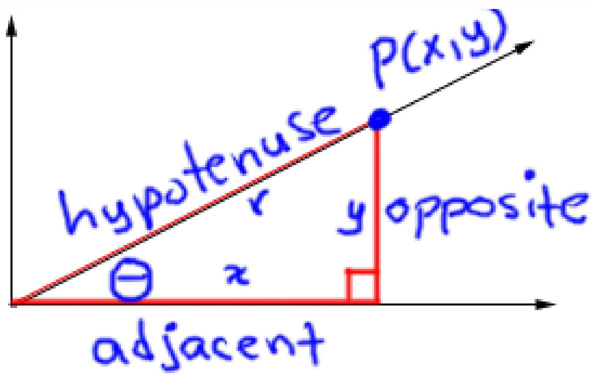
Trig. Standard 9.0: Students compute, by hand, the values of the trigonometric functions and the inverse trigonometric functions at various standard points.
Trig Standard 3.0: Students know the identity $\cos^2(x) + \sin^2(x) = 1$.

Objective: To define trigonometric functions of acute angles.



There are six Trigonometric Ratios in a Right Triangle involving one angle and two sides.

$$\sin\theta = \frac{y}{r} = \frac{Opposite}{Hypotenuse} \qquad \csc\theta = \frac{r}{y} = \frac{Hypotenuse}{Opposite}$$

$$\cos\theta = \frac{x}{r} = \frac{Adjacent}{Hypotenuse} \qquad \sec\theta = \frac{r}{x} = \frac{Hypotenuse}{Adjacent}$$

$$\tan\theta = \frac{y}{x} = \frac{Opposite}{Adjacent} \qquad \cot\theta = \frac{x}{y} = \frac{Adjacent}{Opposite}$$

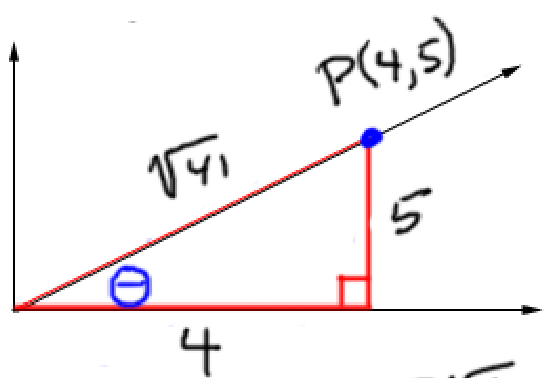
Memory Devices for Sin, Cos, & Tan:

1) SOH CAH TOA
2) ***O***scar ***H***ad ***A*** ***H***eap ***O***f ***A***pples (Sin, Cos, Tan)
3) ***S***addle ***O***ur ***H***orses, ***C***antor ***A***way ***H***appily ***T***owards ***O***ther ***A***dventures

Memory Devices for Csc, Sec, & Cot:

1) ***C***alculators ***H***elps ***O***ur ***S***mart ***H***eads ***A***t ***C***alculating ***A***lgebraic ***O***perations.
2) Csc (at **C**hristmas **S**anta ***C***laus says **HO, HO, HO**)
3) Sec (If you think of the game "Pin The Tail on the Donkey", then Secant is the funniest one: **HA, HA, HA**)
4) Cot (Calypso music has an "**A O**" in it, sing it now...
   "A O, A O daylight come and we want to go home".)

Ex 1) An acute angle θ is in standard position and its terminal side passes through P(4, 5). Find sin θ , cos θ , tan θ , csc θ , sec θ , and cot θ.



Using the Pythagorean Theorem:

$r^2 = 4^2 + 5^2$

$r^2 = 16 + 25$

$r^2 = 41$

$r = \sqrt{41}$

$\sin\theta = \frac{5}{\sqrt{41}} = \frac{5\sqrt{41}}{41}$

$\cos\theta = \frac{4}{\sqrt{41}} = \frac{4\sqrt{41}}{41}$

$\tan\theta = \frac{5}{4}$

$\csc\theta = \frac{\sqrt{41}}{5}$

$\sec\theta = \frac{\sqrt{41}}{4}$

$\cot\theta = \frac{4}{5}$

Definition: A ***Trigonometric Identity*** is an equation involving trigonometric functions of an angle θ that is true for all values of θ.

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{y}{r}}{\frac{x}{r}} = \frac{y}{r}\cdot\frac{r}{x} = \frac{y}{x} = \tan\theta$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{\frac{x}{r}}{\frac{y}{r}} = \frac{x}{r}\cdot\frac{r}{y} = \frac{x}{y} = \cot\theta$$

$$\mathbf{\sin^2\theta + \cos^2\theta = 1}$$

$r^2 = x^2 + y^2$ (Pyth. Thm.)

$$\left(\frac{y}{r}\right)^2 + \left(\frac{x}{r}\right)^2 = 1$$

$$\frac{y^2}{r^2} + \frac{x^2}{r^2} = 1$$

$$\frac{x^2+y^2}{r^2} = 1$$

$$\frac{r^2}{r^2} = 1$$

$$1 = 1$$

Ex 2) Find cos θ and tan θ if θ is an acute angle and sin θ = 1/3.

$$\sin^2\theta + \cos^2\theta = 1$$

$$\left(\frac{1}{3}\right)^2 + \cos^2\theta = 1$$

$$\cos^2\theta = 1 - \left(\frac{1}{3}\right)^2$$

$$\cos^2\theta = 1 - \frac{1}{9}$$

$$\cos^2\theta = \frac{8}{9}$$

$$\sqrt{\cos^2\theta} = \sqrt{\frac{8}{9}}$$

$$\cos\theta = \frac{2\sqrt{2}}{3}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\tan\theta = \frac{\frac{1}{3}}{\frac{2\sqrt{2}}{3}}$$

$$\tan\theta = \frac{1}{3} \cdot \frac{3}{2\sqrt{2}}$$

$$\tan\theta = \frac{1}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$\tan\theta = \frac{\sqrt{2}}{4}$$

Complimentary angles are angles whose sum is 90°.

Cosine means the **Sine** of the **complement**.
Cotangent means the **Tangent** of the **complement.**
Cosecant means the **Secant** of the **complement.**

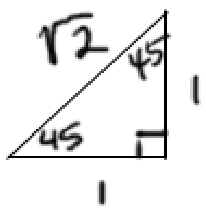
Therefore, we call these functions **Cofunctions**.
Below are the **Cofunction Identities**:

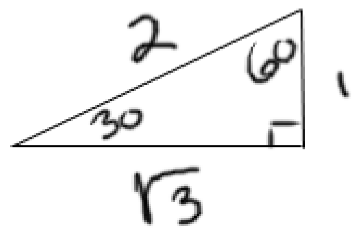
$$\sin\theta = \cos(90-\theta) \qquad \cos\theta = \sin(90-\theta)$$

$$\tan\theta = \cot(90-\theta) \qquad \cot\theta = \tan(90-\theta)$$

$$\sec\theta = \csc(90-\theta) \qquad \csc\theta = \sec(90-\theta)$$

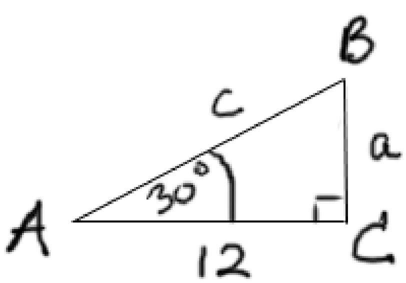
Recall from Geometry two special right triangles:





θ	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$

Ex 3) Use the following diagram to find the lengths of the missing sides.

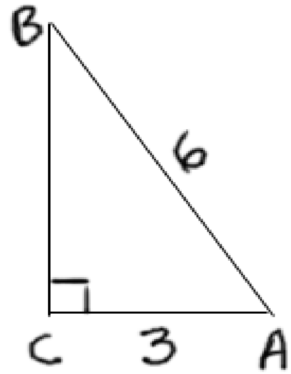


$$\tan 30^\circ = \frac{a}{12}$$
$$\frac{\sqrt{3}}{3} = \frac{a}{12}$$
$$3a = 12\sqrt{3}$$
$$a = 4\sqrt{3}$$

$$\cos 30^\circ = \frac{12}{c}$$
$$\frac{\sqrt{3}}{2} = \frac{12}{c}$$
$$\sqrt{3}c = 24$$
$$c = \frac{24}{\sqrt{3}}$$
$$c = \frac{24}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$
$$c = \frac{24\sqrt{3}}{3}$$
$$c = 8\sqrt{3}$$

Ex 4) Use the diagram to find the measure of $\angle$ A.

$$\cos A = \frac{3}{6}$$
$$\cos A = \frac{1}{2}$$
$$\cos 60 = \frac{1}{2}$$
$$\therefore \angle A = 60^\circ$$



Another way to remember some Trig. values:

Sin 0 $= \frac{\sqrt{0}}{2} = 0$ — Cos 90

Sin 30 $= \frac{\sqrt{1}}{2} = \frac{1}{2}$ — Cos 60

Sin 45 $= \frac{\sqrt{2}}{2}$ — Cos 45

Sin 60 $= \frac{\sqrt{3}}{2}$ — Cos 30

Sin 90 $= \frac{\sqrt{4}}{2} = \frac{2}{2} = 1$ — Cos 0

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$