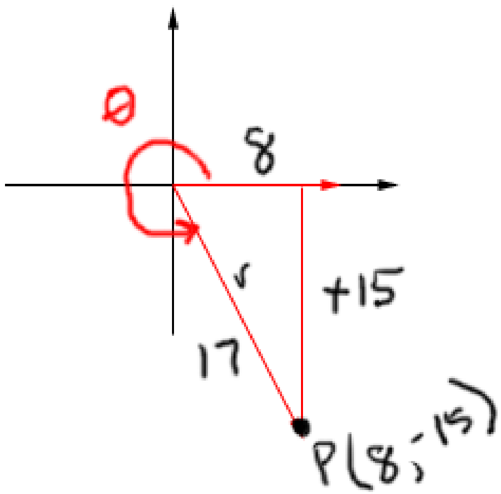
12-3 Trigonometric Functions of General Angles Page 561

Objective: To define trigonometric functions of general angles.

To define trigonometric angles greater than 90 degrees, start in standard position, find the point P(x, y) on the terminal side in whatever quadrant it may be, and let ***r*** represent the distance from the origin to point P.

Ex 1) Find the values of all 6 trigonometric functions of an angle θ in standard position whose terminal side passes through the point (8, -15).



$r^2 = 8^2 + (-15)^2$

$r^2 = 64 + 225$

$r^2 = 289$

$\sqrt{r^2} = \sqrt{289}$

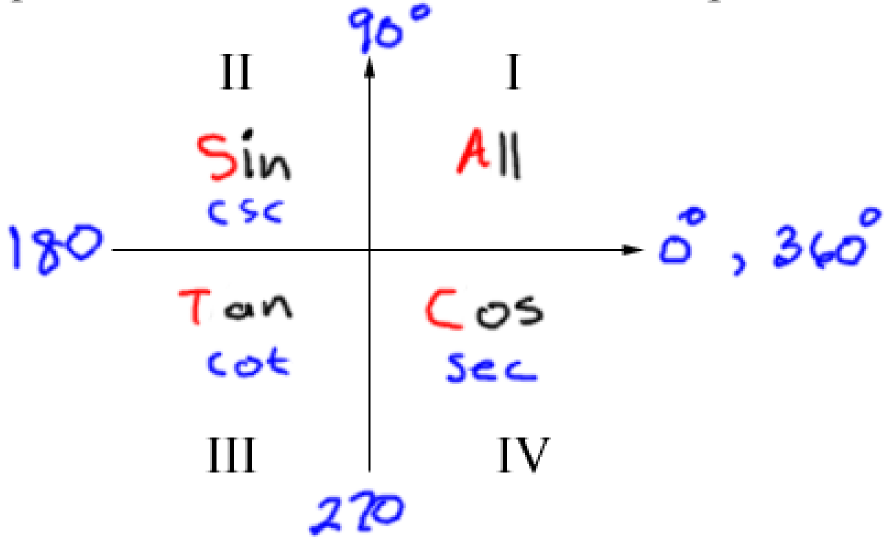
$r = 17$

$\sin\theta = \frac{y}{r} = \frac{-15}{17}$ $\qquad$ $\csc\theta = \frac{r}{y} = \frac{+17}{-15} = \frac{-17}{15}$

$\cos\theta = \frac{x}{r} = \frac{8}{17}$ $\qquad$ $\sec\theta = \frac{r}{x} = \frac{17}{8}$

$\tan\theta = \frac{y}{x} = \frac{-15}{8}$ $\qquad$ $\cot\theta = \frac{x}{y} = \frac{8}{-15} = \frac{-8}{15}$

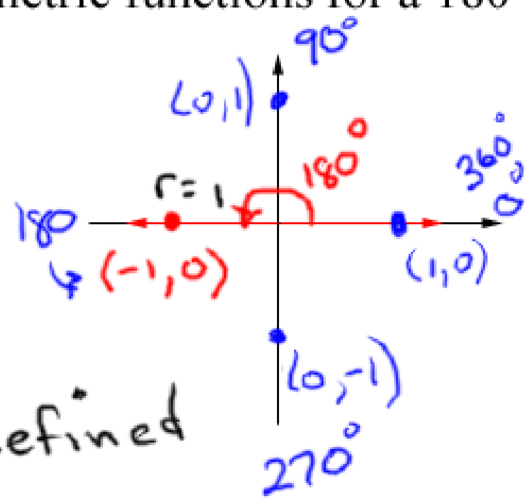
Different Trigonometric functions are positive in different quadrants. The list below can help:



Here's a saying that can help you remember this:
***A**ll **S**tudents **T**ake **C**alculus*

Don't forget that these functions have reciprocals!

Ex 2) Find the values of all trigonometric functions for a 180° angle.



$\sin 180° = \frac{y}{r} = \frac{0}{1} = 0$

$\cos 180° = \frac{x}{r} = \frac{-1}{1} = -1$

$\tan 180° = \frac{y}{x} = \frac{0}{-1} = 0$

$\csc 180° = \frac{r}{y} = \frac{1}{0} = \text{undefined}$

$\sec 180° = \frac{r}{x} = \frac{1}{-1} = -1$

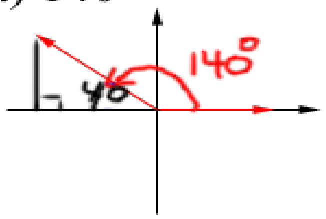
$\cot 180° = \frac{x}{y} = \frac{-1}{0} = \text{undefined}$

Angles in standard position larger than 90° that are not quadrantal angles, have a unique acute angle associated with it called a **reference angle**.

A **reference angle** is an acute angle formed by the terminal side and the *x*-axis.

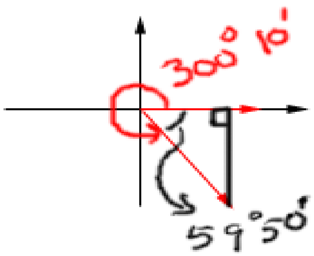
Ex 3) Find the reference angle for each angle listed below:

3a) 140°



$180° - 140° = 40°$

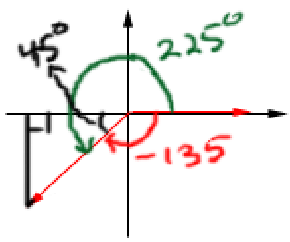
3b) 300° 10'



$360° - 300°10' = 59°50'$

$$\begin{array}{r} 359° \ 60' \\ \cancel{360°} \\ 300° \ 10' \\ \hline 59° \ 50' \end{array}$$

3c) -135°



$-135 + 360 = 225°$

$225° - 180° = 45°$

Ex 4) Write cos 210° as a function of an acute angle.

$210 - 180 = 30°$

cosine is negative in Quadrant III

$\therefore \cos 210° = -\cos 30°$

$= -\frac{\sqrt{3}}{2}$

Ex 5) Find the exact value of the following:

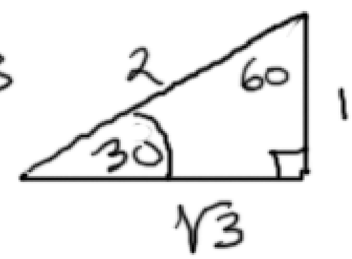
5a) tan 330°

reference $\angle = 360° - 330° = 30°$

tangent is negative in Quadrant IV

$\tan 330° = -\tan 30° = -\frac{1}{\sqrt{3}}$

$= -\frac{\sqrt{3}}{3}$



5b) csc (-225°)

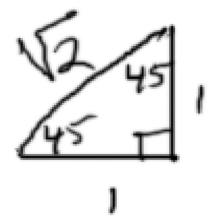
$-225° + 360° = 135°$

referance $\angle$ for $135° = 45°$ $(180° - 135°)$

csc is positive in Quadrant II

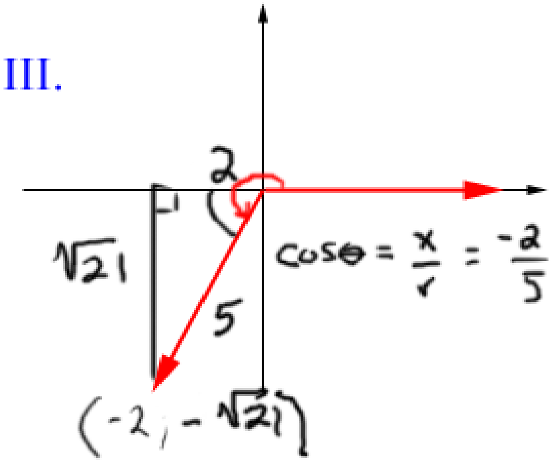
$\therefore \csc(-225°) = \csc 45°$

$= \sqrt{2}$



Ex 6) Find sin θ and tan θ if cos θ = -2/5 and 180° < θ < 360°
(θ is in quadrants III or IV).

Cosine is positive in Quadrant IV
so this angle must be in Quadrant III.



$$5^2 = y^2 + (-2)^2$$
$$25 = y^2 + 4$$
$$y^2 = 21$$
$$\sqrt{y^2} = \sqrt{21}$$
$$y = \sqrt{21}$$

$\sin\theta = -\dfrac{\sqrt{21}}{5}$ (sin is negative in Q III)

$\tan\theta = \dfrac{-\sqrt{21}}{-2} = \dfrac{\sqrt{21}}{2}$ (tan is positive in Q III)

Reference Angles