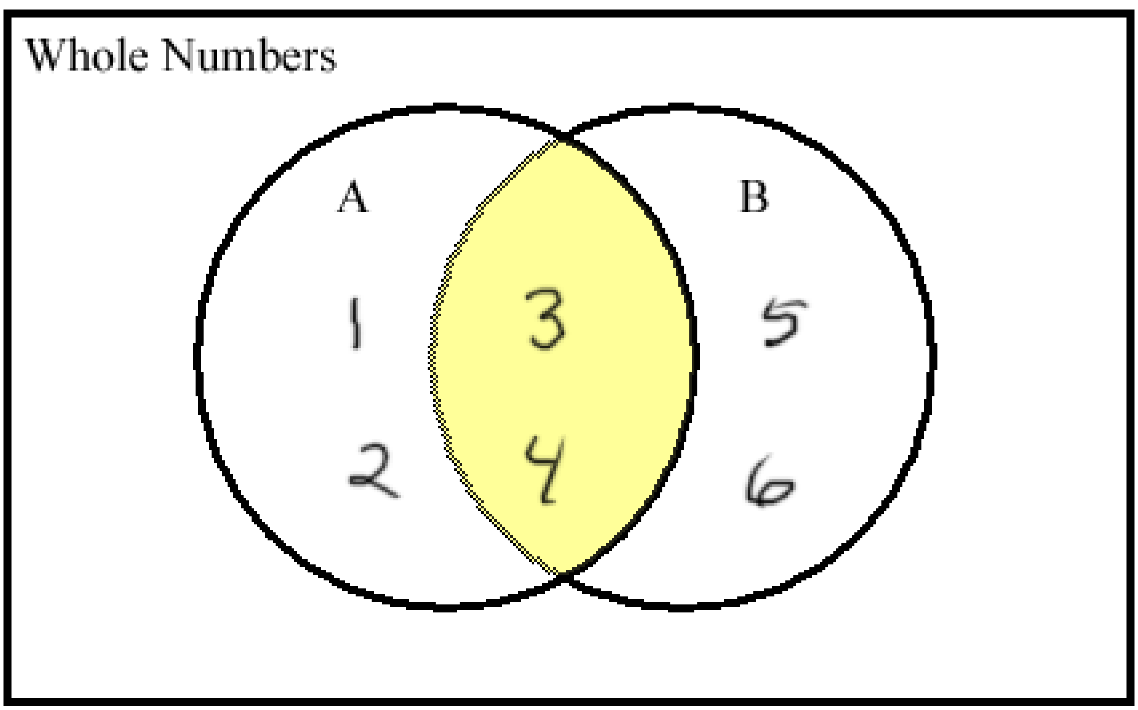
Alg 2 15-10 Mutually Exclusive and Independent Events Page 754

Algebra 2 Probability Standard #1.0
Students know the definition of the notion of independent events and can use the rules for addition, multiplication, and complementation to solve for probabilities of particular events in finite sample spaces.

Objective: To identify mutually exclusive and independent events and find the probability of such events.

Let's look at the following two sets A, and B:
A = { 1, 2, 3, 4} and B ={3, 4, 5, 6}
...they have the numbers 3 and 4 in common.

Here is a Venn diagram representation of the two sets.



The Intersection of ***A*** and ***B*** is
$\{1, 2, 3, 4\} \cap \{3, 4, 5, 6\} = \{3, 4\}$

Some sets have nothing in common. Consider the intersection of the following sets:

$\{1, 2, 3\} \cap \{4, 5, 6\} = \emptyset$

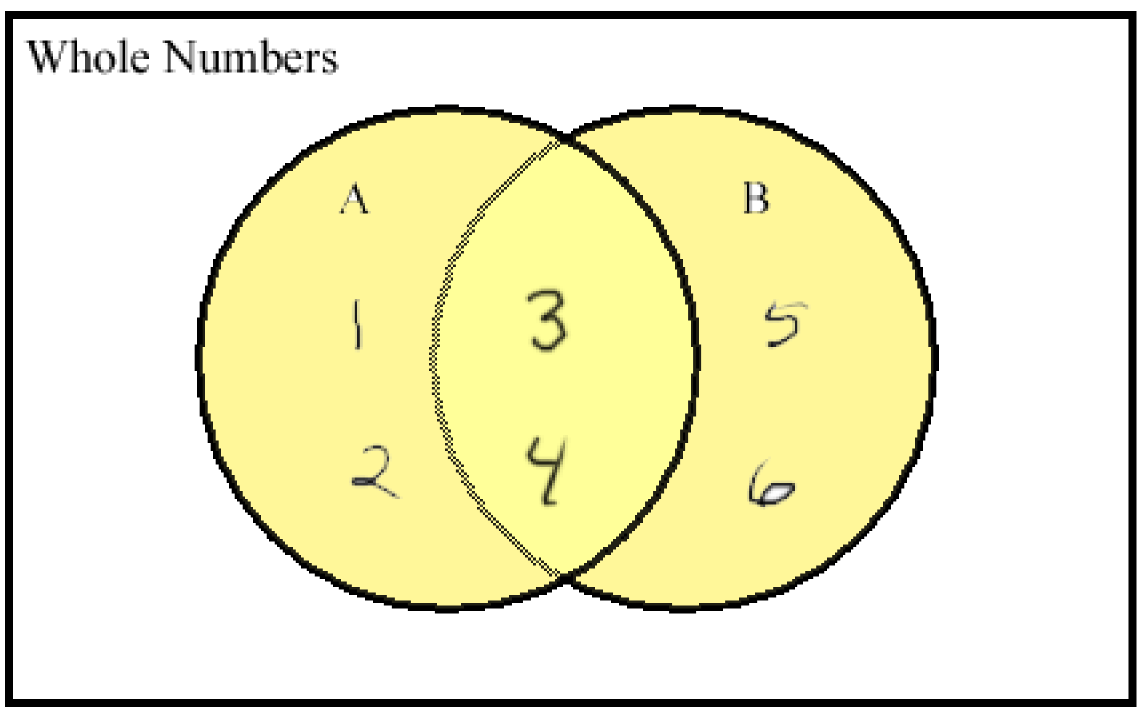
Sets with nothing in common are called **disjoint** sets.

The union of two sets ***A*** and ***B*** consists of all the elements of ***A*** together with all of the elements of ***B***, for example:

If ***A*** = {1, 2, 3, 4} and ***B*** = {3, 4, 5, 6}, the union of the sets is the set {1, 2, 3, 4, 5, 6}.

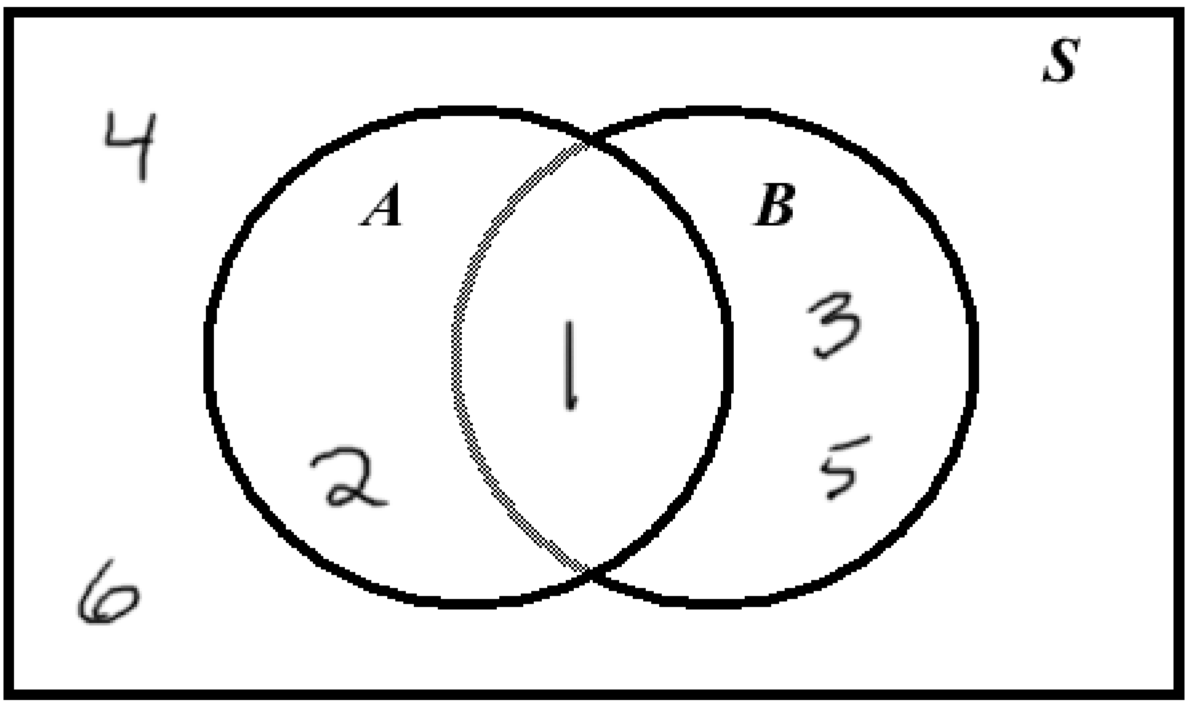
The following notation represents the union of the two sets and is illustrated by the Venn diagram:

$A \cup B$



The Venn diagram below shows the sample space ***S*** for the experiment of drawing a number at random from the set {1, 2, 3, 4, 5, 6}. Every subset of ***S*** is an **event**, including ***S*** itself and the empty set, Ø. These two events have a probability of 1 and 0 respectively.

For any sample space *S*, P(S) = 1 and P(Ø) = 0.



The Venn diagram shows events ***A*** and ***B*** in the sample space ***S***. Event ***A***, {1, 2} is the drawing of an number less than three, while event ***B***, {1, 3, 5} is the drawing of an odd number.

$$P(A)=\frac{2}{6} \text{ and } P(B)=\frac{3}{6}$$

The probability that either ***A*** or ***B*** occurs is P(A ∪ B).
Since A ∪ B = {1, 2, 3, 5}, P(A ∪ B) = 4/6.

$$P(A \cup B) = \frac{4}{6} = \frac{2}{3} + \frac{3}{6} - \frac{1}{6} =$$
$$P(A) + P(B) - P(A \cap B)$$

For any two events *A* and *B* in a sample space,
P(A ∪ B) = P(A) + P(B) - P(A ∩ B)

If two events have no elements in common, (their intersection is empty), the events are called **mutually exclusive** and **P(A ∩ B) = P(Ø) = 0.**

If ***A*** and ***B*** are **mutually exclusive events** then,
P(A ∪ B) = P(A) + P(B).

Ex 2) From a group of 10 seniors and 8 juniors, 3 students are to be selected at random to form a committee. What is the probability that at least 2 seniors are selected?

Solution: The selection will include at least 2 seniors if either exactly 2 seniors and 1 junior are selected (event ***M***) or 3 seniors are selected (event ***N***).
Since these two events are mutually exclusive:
P(M ∪ N) = P(M) + P(N).

$$P(m) = \frac{{}_{10}C_2 \cdot {}_8C_1}{{}_{18}C_3} = \frac{45 \cdot 8}{816} =$$

$$\frac{360}{816} = \frac{15}{34}$$

$$P(N) = \frac{{}_{10}C_3}{{}_{18}C_3} = \frac{120}{816} = \frac{5}{34}$$

$$P(m \cup N) = \frac{15}{34} + \frac{5}{34} = \frac{20}{34} = \frac{10}{17}$$

If the occurence of one event has no effect on the probability of another event, the two events are said to be **independent**.

The events *A* and *B* are independent if and only if: P(A ∩ B) = P(A) · P(B).

Ex 3) A bag contains 4 green and 4 blue buttons.
Two buttons are drawn at random from the bag as follows:

a) The first button drawn is put back into the bag before the second button is drawn.

b) The first button drawn is ***not*** put back into the bag before the second button is drawn.

Let ***A*** be the event that the first button drawn is green.
Let ***B*** be the event that the second button drawn is blue.

a) $P(A) = \frac{4}{8} = \frac{1}{2}$

$P(B) = \frac{4}{8} = \frac{1}{2}$

$$P(A \cap B) = \frac{{}_4C_1 \cdot {}_4C_1}{{}_8C_1 \cdot {}_8C_1} = \frac{4 \cdot 4}{8 \cdot 8} = \frac{16}{64} = \frac{1}{4}$$

$$P(A) \cdot P(B) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} = P(A \cap B)$$

Therefore, events ***A*** and ***B*** **are independent**. (Since the first button was put back, the result of **the first draw *doesn't* affect the outcome of the second draw**.)

b) $P(A) = \frac{1}{2}$

Event ***B*** can occur in two mutually exclusive ways.
1) You either draw a green button first and then draw one of the four blue buttons from the seven remaining, or 2) you draw a blue button first and then draw one of the other three blue buttons from the seven remaining.
P(B)=P(first green, then blue) + P(first blue, then blue)

$$= \frac{4}{8} \cdot \frac{4}{7} + \frac{4}{8} \cdot \frac{3}{7}$$

$$= \frac{16}{56} + \frac{12}{56} = \frac{28}{56} = \frac{1}{2}$$

The event A ∩ B is equivalent to drawing a green button first followed by a blue button.
So, P(A ∩ B) = P(green first, then blue) =

$$\frac{4}{8} \cdot \frac{4}{7} = \frac{16}{56} = \frac{2}{7}$$

$P(A) \cdot P(B) = 1/2 \cdot 1/2 = 1/4 \neq P(A \cap B)$

Therefore, events *A* and *B* are dependent, which means that the result of the first draw affects the next draw.

The **complement** of an event ***A***, consists of the elements in ***S*** that are **not** members of the event ***A***.

The symbol for the complement of event ***A*** is $\bar{A}$ and the probability of $\mathbf{P(\bar{A}) = 1 - P(A)}$.

Ex 4) Based on past performances in the school band, the probability that Ryan will be selected to perform in the county band is 3/4, that Faye will be selected is 2/3, and that Chung will be selected is 1/2. Suppose that the selection of one student does not affect another student's chances. Find the probabilities of the following events:

a) Ryan and Chung are selected, but Faye is not.

b) At least one of the three is selected.

Let ***R*** be the event that Ryan is selected for the county band, ***F*** is the event that Faye is selected, and ***C*** be the event that Chung is selected. Then:

$$P(R) = \frac{3}{4}, \quad P(F) = \frac{2}{3}, \quad P(C) = \frac{1}{2}$$

a) The probability that Faye is *not* selected is:

$$P(\bar{F}) = 1 - P(F) = 1 - \frac{2}{3} = \frac{1}{3}$$

The probability that Ryan and Chung are selected, but Faye is not selected:

$$P(R) \cdot P(C) \cdot P(\bar{F}) = \frac{3}{4} \cdot \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{8}$$

b) Let ***D*** be the event that *at least one* of the three studets is selected, and let ***E*** be the event that *none* of them is selected. Note that D = Ē and P(D) = P(Ē).

$$P(D) = P(\bar{E})$$
$$= 1 - P(E)$$
$$= 1 - P(\bar{R}) \cdot P(\bar{F}) \cdot P(\bar{C})$$
$$= 1 - \frac{1}{4} \cdot \frac{1}{3} \cdot \frac{1}{2}$$
$$= 1 - \frac{1}{24} = \frac{23}{24}$$