Let's take a look at some pizza possibilities
(Run the program pizza.exe)

In situations where we consider combinations of items, or a succession of events such as flips of a coin, or the drawing of cards, or choosing pizza crusts, toppings, or meats, each result is called an outcome.

An event is a subset of outcomes (one draw of the card, one flip of the coin, or the choice of a pizza crust).

When several events occur together, such as choosing a card followed by choosing another card, or choosing a pizza crust followed by choosing a pizza topping, we have a compound event.

The Fundamental Counting Principle
In a compound event in which the first event may occur in \(n_1\) ways, the second event may occur in \(n_2\) ways, etc. The \(k^{th}\) event may occur in \(n_k\) different ways, so the total number of ways the compound event may occur is:

\[n_1 \cdot n_2 \cdot n_3 \cdot \ldots \cdot n_k\]
Ex 1) How many odd 2-digit whole numbers are there that are less than 70?

\[
\begin{array}{c}
\text{tens digit} \\
1, 2, 3, 4, 5, 6
\end{array}
\times
\begin{array}{c}
\text{ones digit} \\
1, 3, 5, 7, 9
\end{array} = 30
\]

To get to school, you can either walk or arrive by car. These are called **mutually exclusive** choices. You can do one or the other but not both at the same time.

If the possibilities being counted can be grouped into **mutually exclusive** cases, then the number of possibilities is the **sum** of the number of possibilities in each case.

Ex 2) How many positive integers less than 100 can be written using the digits 6, 7, 8, and 9?
There are two mutually exclusive cases here:
1) one digit integers, and
2) the two digit integers.

1) The one digit case:
\[
\begin{array}{c}
\text{ones digit} \\
\hline
6,7,8,9
\end{array}
\begin{array}{c}
4 \\
\hline
= 4
\end{array}
\]

2) The two digit case:
\[
\begin{array}{c}
tens \\
\hline
6,7,8,9
\end{array}
\begin{array}{c}
4 \\
\hline
\end{array}
\begin{array}{c}
\times
\\
\hline
\end{array}
\begin{array}{c}
\text{ones} \\
\hline
6,7,8,9
\end{array}
\begin{array}{c}
4 \\
\hline
= 16
\end{array}
\]
\[
\begin{array}{c}
\hline
20
\end{array}
\]
Ex 3) How many ID tags can be made using 3 symbols (letters and digits) with at least one letter in each?

1) one letter case:
\[
\frac{26 \cdot 10 \cdot 10}{9-2 \ 0-9 \ 0-9} = 2600
\]

The letter can occur in any of the three places, so there are: \(3 \cdot 2600 = 7800\) possible 1 letter tags.

2) two letter case:
\[
\frac{26 \cdot 26 \cdot 10}{a-z \ a-z \ 0-9} = 6760
\]

The digit can occur in any of the three places, so there are: \(3 \cdot 6760 = 20,280\) possible 2 letter tags.

3) three letter case:
\[
\frac{26 \cdot 26 \cdot 26}{a-z \ a-z \ a-z} = 17576
\]

There are 17,576 possible 3 letter tags.

The total number of tags is the sum of the 3 mutually exclusive possibilities: \(7800 + 20,280 + 17,576 = 45,656\)