Consider a set of three objects \{a, b, c\}.
How many ways are there to "order" or "arrange" these objects?

\begin{align*}
\text{abc} & \quad \text{bac} \quad \text{cab} \\
\text{acb} & \quad \text{bca} \quad \text{cba}
\end{align*}

Each of these arrangements is called a permutation of the letters a, b, and c.

A permutation of a set of objects is an ordered arrangement of the objects. (Keywords: order, arrange)

To determine the number of permutations of the letters a, b, and c without listing them, we can use the Fundamental Counting Principle that we learned in lesson 15-5.

\[
\frac{3 \cdot 2 \cdot 1}{1st \quad 2nd \quad 3rd} = 6 \text{ arrangements}
\]

Ex 1) Find the number of permutations of the four letters: p, q, r, and s.

\[
\frac{4 \cdot 3 \cdot 2 \cdot 1}{1st \quad 2nd \quad 3rd \quad 4th} = 24
\]
The number of permutations of $n$ objects is $n!$
(There are $n$ objects to choose from and we arrange all of them)

$$nP_n = n!$$

Reminder: $n! = n\cdot(n-1)\cdot(n-2)\cdot\ldots\cdot 3 \cdot 2 \cdot 1$

Ex 2) In how many ways can the letters in the word: JUSTICE be arranged using only 5 letters at a time?

$$\frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3}{1st \quad 2nd \quad 3rd \quad 4th \quad 5th} = \boxed{2520}$$

The number of permutations of a set of $n$ objects taken $r$ at a time is given in the following formula:

$$nP_r = \frac{n!}{(n-r)!}$$

If you have this button on your calculator, you can use it...
Ex 3) From a set of 9 books, 4 are to be selected and arranged on a book shelf. How many arrangements are possible?

$$\binom{9}{4} = \frac{9!}{(9-4)!} = \frac{9\cdot8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1}{5\cdot4\cdot3\cdot2\cdot1} = 3024$$

If a set of \( n \) elements has \( n_1 \) elements of one kind alike, \( n_2 \) of another kind alike, and so on, then the number of permutations, \( P \), of the \( n \) elements taken \( n \) at a time is given by the formula:

$$P = \frac{n!}{n_1!n_2!...}$$

Ex 4) Find the number of ways the letters of the word: HUBBUBUB can be arranged.

There are 6 letters. There are 2 U's and 3 B's. Therefore, we use the formula:

$$P = \frac{n!}{n_1!n_2!...} = \frac{6!}{2!3!} = \frac{6\cdot5\cdot4\cdot3\cdot2\cdot1}{(2\cdot1)\cdot(3\cdot2\cdot1)} = 60$$