15-6 Permutations Page 734

Consider a set of three objects {a, b, c}.
How many ways are there to "order" or "arrange" these objects?

abc bac cab

acb bca cba

Each of these arrangements is called a **permutation** of the letters a, b, and c.

A **permutation** of a set of objects is an ***ordered arrangement*** of the objects. (Keywords: order, arrange)

To determine the number of permutations of the letters a, b, and c without listing them, we can use the Fundamental Counting Principle that we learned in lesson 15-5.

$\underset{\text{1st}}{\underline{\;3\;}} \cdot \underset{\text{2nd}}{\underline{\;2\;}} \cdot \underset{\text{3rd}}{\underline{\;1\;}} = 6$ arrangements

Ex 1) Find the number of permutations of the four letters: p, q, r, and s.

$\underset{\text{1st}}{\underline{\;4\;}} \cdot \underset{\text{2nd}}{\underline{\;3\;}} \cdot \underset{\text{3rd}}{\underline{\;2\;}} \cdot \underset{\text{4th}}{\underline{\;1\;}} = 24$

The number of permutations of *n* objects is *n*!
(There are *n* objects to choose from and we arrange **all** of them)

$${}_nP_n = n!$$

Reminder: n! = n·(n-1)·(n-2)· ...· 3 · 2 · 1

Ex 2) In how many ways can the letters in the word:
JUSTICE be arranged using only 5 letters at a time?

$$\underset{\text{1st}}{\underline{7}} \cdot \underset{\text{2nd}}{\underline{6}} \cdot \underset{\text{3rd}}{\underline{5}} \cdot \underset{\text{4th}}{\underline{4}} \cdot \underset{\text{5th}}{\underline{3}} = 2520$$

The number of permutations of a set of *n* objects taken *r* at a time is given in the following formula:

$${}_nP_r = \frac{n!}{(n-r)!}$$

If you have this button on your calculator, you can use it...

Ex 3) From a set of 9 books, 4 are to be selected and **arranged** on a book shelf. How many **arrangements** are possible?

$$_9P_4 = \frac{9!}{(9-4)!} = \frac{9 \bullet 8 \bullet 7 \bullet 6 \bullet \cancel{5} \bullet \cancel{4} \bullet \cancel{3} \bullet \cancel{2} \bullet \cancel{1}}{\cancel{5} \bullet \cancel{4} \bullet \cancel{3} \bullet \cancel{2} \bullet \cancel{1}}$$

$$= 3024$$

If a set of n elements has n_1 elements of one kind alike, n_2 of another kind alike, and so on, then the number of permutations, P, of the n elements taken n at a time is given by the formula:

$$P = \frac{n!}{n_1!n_2!\ldots}$$

Ex 4) Find the number of ways the letters of the word: HUBBUB can be arranged.

There are 6 letters. There are 2 U's and 3 B's. Therefore, we use the formula:

$$P = \frac{n!}{n_1!n_2!\ldots} = \frac{6!}{2!3!} = \frac{6 \bullet 5 \bullet \overset{2}{\cancel{4}} \bullet \cancel{3} \bullet \cancel{2} \bullet \cancel{1}}{(\cancel{2} \bullet 1) \bullet (\cancel{3} \bullet 2 \bullet \cancel{1})} = 60$$