Reminder: Set B is a subset of A if each member of the set B is also a member of A. The empty set (null set, ∅, { }) is considered to be a subset of any set, and the entire set is considered to be a subset of itself as well.

Ex 1) For the three letter set {P, Q, R}

1a) Find all the subsets.

1a) {P, Q, R}, {P, Q}, {P, R}, {Q, R}, {P}, {Q}, {R}, ∅

1b) Find all the 2 letter subsets.

1b) {P, Q}, {P, R}, {Q, R}

These subsets are also known as ***combinations*** of the letters P, Q, and R. (Keywords: subset, choose, select)

The number of **combinations** of a set of ***n*** objects taken ***r*** at a time is:

$$\binom{n}{r} = {}_nC_r = \frac{n!}{r!(n-r)!}$$

Ex 2) Find the number of combinations of the letters in the word SOLVE, taking them (a) 5 at a time and (b) 2 at a time. List each combination.

2a) {S, O, L, V, E}

$$\binom{5}{5} = {}_5C_5 = \frac{5!}{5!(5-5)!} = \frac{5!}{5!0!} = 1$$

2b) {S, O}, {S, L}, {S, V}, {S, E}, {O, L},
{O, V}, {O, E}, {L, V}, {L, E}, {V, E}.

$$\binom{5}{2} = {}_5C_2 = \frac{5!}{2!(5-2)!} = \frac{5!}{2!3!} = \frac{5 \cdot 4 \cdot 3!}{(2 \cdot 1) \cdot 3!}$$

$$= 10$$

Ex 3) In how many ways can a committee of 6 be **chosen** from 5 teachers and 4 students if:

a) all people are equally eligible?

b) the committee must include 3 teachers and 3 students?

a) There are 9 people eligible for the committee and we will **choose** 6.

$$\binom{9}{6} = {}_9C_6 = \frac{9!}{6!(9-6)!} = \frac{9!}{6!3!} = \frac{9 \cdot 8 \cdot 7 \cdot 6!}{6! \cdot (3 \cdot 2 \cdot 1)}$$

$$= 84$$

3b) This is a *compound event* (section 15-5) where we **choose** 3 teachers from a list of 5 AND **choose** 3 students from a list of 4. (Hint: Find the number of combinations in each case and multiply the results)

$$ {}_5C_3 \bullet {}_4C_3 = \frac{5!}{3!(5-3)!} \bullet \frac{4!}{3!(4-3)!} = \frac{5!}{3!2!} \bullet \frac{4!}{3!1!} $$

$$ = \frac{5 \cdot \overset{2}{\cancel{4}} \cdot \cancel{3!}}{\cancel{3!}(\cancel{2} \cdot 1)} \bullet \frac{4 \cdot \cancel{3!}}{\cancel{3!} \cdot 1} $$

$$ = 10 \cdot 4 $$

$$ = \boxed{40} $$

Ex 4) A standard deck of cards consists of 4 suits (clubs, diamonds, hearts, spades) of 13 cards each. How many 5-card hands can be dealt that include 4 cards from the same suit and one card from a different suit?

There are 4 suits with "13 choose 4" possible combinations from any one suit.

$$4\left({}_{13}C_4\right) = 4\left(\frac{13!}{4!(13-4)!}\right) = 4\left(\frac{13!}{4!\,9!}\right)$$

$$= 4\left(\frac{13\cdot 12\cdot 11\cdot 10\cdot 9!}{(4\cdot 3\cdot 2\cdot 1)\,9!}\right)$$

$$= 4(715)$$

$$= 2860$$

The remaining card must come from the other 3 suits... This leaves 52 cards minus 13 cards (from one suit) 52 - 13 = 39 cards.

$${}_{39}C_1 = \frac{39!}{1!\,38!} = \frac{39\cdot 38!}{38!\cdot 1} = 39$$

The number of 5-card hands that include exactly 4 cards from any suit is 2860 • 39 = 111,540.