Algebra 2 15-8 Sample Spaces and Events Page 743

Algebra 2 Probability State Standards
1.0 Students know the definition of the notion of independent events and can use the rules for addition, multiplication, and complementation to solve for probabilities of particular events in finite sample spaces.
2.0 Students know the definition of conditional probability and use it to solve for probabilities in finite sample spaces.

Objective: To specify sample spaces and events for random experiments.

Definitions:

Random Experiment: an experiment where you don't necessarily get the same outcome when you repeat the experiment under the same conditions
(i.e. flipping a coin, rolling a die, choosing a card from a deck)

Sample space: The set of all possible outcomes for the random experiment. (i.e. Heads or Tails for the coin: sample space = {H,T}; the numbers 1 through 6 for the roll of a standard die: sample space = {1, 2, 3, 4, 5, 6})

Event: Any subset of the possible outcomes for an experiment.

Simple Event: When an event has a single element in it's sample space (i.e. there is only one outcome: a coin toss has only Heads OR Tails)

Ex. 1) When rolling a single die, specify:
a) the sample space for the experiment
b) the event that a number less than 4 result
c) the event that an even number results.

a) Sample space: {1, 2, 3, 4, 5, 6}

b) number less than 4: {1, 2, 3}

c) an even number results: {2, 4, 6}

The sample space for rolling 2 dice can be thought of as a compound event with 6 outcomes for the first die and 6 outcomes for the second die. Remember that for compound events we use the Fundamental Counting Principle and multiply the outcomes. So the sample space for 2 dice is calculated by $6 \cdot 6 = 36$ elements.

The outcome for rolling 2 dice can be represented using ordered pairs, where the first coordinate represents the outcome for the first die and the second coordinate represents the outcome for the second die. For example, (5, 2) represents a "roll" of 5 for the first die and a 2 for the second die.

If we represent the outcome of rolling of two dice as an ordered pair, where the first die is represented by the first number in the ordered pair, and the second die is represented by the second number in the ordered pair, then the sample space is as follows:

{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),
(2,1), (2,2), (2,3), (2,4), (2,5), (2,6),
(3,1), (3,2), (3,3), (3,4), (3,5), (3,6),
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6),
(5,1), (5,2), (5,3), (5,4), (5,5), (5,6),
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)}

There are 36 possible outcomes.

Ex 2) When rolling 2 dice, specify each event:
a) The sum of the numbers of the dice is 5
b) The first die rolls a 3

a) {(1, 4), (2, 3), (3, 2), (4,1)}

b) {(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)}

Probability is the measure of how likely an event is to occur.

Equally likely outcomes have the same chance of occurring. (flipping a coin: Heads and Tails are equally likely)

We will be working with **theoretical probability** instead of **experimental probability**.

For equally likely outcomes (in theoretical probability), the probability that an event may occur is the ratio of the *favorable* (or *desired*) outcomes to the total number of outcomes.

$$P(event) = \frac{\text{number of favorable outcomes}}{\text{number of outcomes in the sample space}}$$

Some books say the the ***desired*** or ***favorable*** outcome is a ***success*** and any other outcome is a ***failure***.

Using these terms, if an event can succeed in ***s*** ways and can fail in ***f*** ways, then the probability of success, P(***s***), and of failure, P(***f***) are noted below.

$$P(s) = \frac{s}{s+f} \qquad P(f) = \frac{f}{s+f}$$

Note: The total number of outcomes is equal to the sum of the number of successes and failures.

Probabilities can be written as ratios (fractions), decimals, or percents. Fractions and decimals are greater than or equal to 0 (zero) AND less than or equal to 1 (one). Percents are greater than or equal to 0% AND less than or equal to 100%.

$$0 \le P(E) \le 1$$

If the probability of the event is zero, P(E) = 0, then the event is **impossible**. If the probability of the event is one, P(E) = 1, then the event is **certain**.
If the probability of two different events are the same, then the events are equally likely. $P(E_1) = P(E_2)$

Ex 3) A music CD has 5 songs that your favorite songs on the CD, and 7 songs that you don't like as much. What is the probability that a randomly selected song is one of your favorites?

There are 12 possible outcomes (12 songs), and 5 favorable or desired outcomes (your 5 favorite songs).

$$P(\text{fav song}) = \frac{5}{12} = 0.417 = 41.7\%$$

Ex 4) A bag contains 4 red marbles, 6 blue marbles, 3 yellow marbles, and 2 green marbles. If one marble is chosen at random, what is the probability that it is blue?

There are 15 possible outcomes (15 marbles in the bag), and 6 favorable or desired outcomes (6 blue marbles).

$$P(\text{blue marble}) = \frac{6}{15} = 0.4 = 40\%$$