Alg 2 15-9 Probability

Algebra 2 Probability State Standards
1.0 Students know the definition of the notion of independent events and can use the rules for addition, multiplication, and complementation to solve for probabilities of particular events in finite sample spaces.
2.0: Students know the definition of

probability and use it to solve for probabilities in finite sample spaces.

Objective: To find the probability that an event will occur.

When flipping a coin, the sample space contains two elements {H, T}, these two outcomes are ***equally likely*** to occur. Therefore, the probabillity of event {H}, denoted by P(H) is equal to the probability of event {T}, denoted by P(T). Or, $P(H) = P(T) = \frac{1}{2}$.

When rolling a fair die, there are 6 possible outcomes, with a sample space of {1, 2, 3, 4, 5, 6}. Each of the six simple outcomes is equally likely, therefore, $P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = 1/6$.

In general, if $\{a_1, a_2, a_3, \ldots a_n\}$ is a sample space containing n equally likely outcomes, then the probability of each simple event is $\frac{1}{n}$:

$$P(a_1) = P(a_2) = P(a_3) = \ldots = P(a_n) = \frac{1}{n}$$

When rolling a die, suppose ***A*** is the event that the outcome is an odd number. Event ***A*** the subset {1, 3, 5} of the sample space {1, 2, 3, 4, 5, 6}. The probability of ***A*** is defined as the **sum** of the probabilities of the simple events that make up ***A*.**

Therefore, $P(A) = P(1) + P(3) + P(5)$

$$= \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$$

In general, if the sample space for an experiment consists of n equally likely outcomes, and if k of them are in the event E, then:

$$P(E) = \underbrace{\frac{1}{n} + \frac{1}{n} + \frac{1}{n} + \ldots + \frac{1}{n}}_{K \text{ addends}} = \frac{k}{n}$$

If the event E contains all of the elements of the sample space, then $P(E) = \frac{n}{n} = 1.$

Therefore, the event is certain to occur.

If the event E contains none of the elements of the sample space, then $P(E) = \frac{0}{n} = 0.$

Therefore, the event is certain *not* to occur.

$$0 \leq P(E) \leq 1$$

Ex 1. A die is rolled. Find the probability of each event:
a) Event A: The number showing is less than 5.
b) Event B: The number showing is between 2 and 6.

a) Event $A = \{1, 2, 3, 4\}$

$$\therefore P(A) = P(1) + P(2) + P(3) + P(4) = \frac{4}{6} = \frac{2}{3}$$

b) Event B $= \{3, 4, 5\}$

$$\therefore P(B) = P(3) + P(4) + P(5) = \frac{3}{6} = \frac{1}{2}$$

Ex. 2 Two dice are rolled and the numbers are noted. Find the probability of each event:
a) Event A: The sum of the numbers is less than 5.
b) Event B: The sum of the numbers is 4 or 5.

If we represent the outcome of rolling of two dice as an ordered pair, where the first die is represented by the first number in the ordered pair, and the second die is represented by the second number in the ordered pair, then the sample space is as follows:

{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),
(2,1), (2,2), (2,3), (2,4), (2,5), (2,6),
(3,1), (3,2), (3,3), (3,4), (3,5), (3,6),
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6),
(5,1), (5,2), (5,3), (5,4), (5,5), (5,6),
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)}

There are 36 possible outcomes.

a) Event A = {(1,1), (1,2), (1,3), (2,1), (2,2), (3,1)}

$$\therefore P\left(A\right) = \frac{6}{36} = \frac{1}{6}$$

b) Event B = {(1,3), (1,4), (2,2), (2,3), (3,1), (3,2), (4,1)}

$$\therefore P\left(B\right) = \frac{7}{36}$$

Ex. 3 There are 12 tulip bulbs in a package. Nine will yield yellow tulips and three will yield red tulips. If two tulip bulbs are selected at random, find the probability of each event:

a) Event Q: Both tulip bulbs will be red.
b) Event R: One tulip will be yellow and the other will be red.

a) Since the bulbs are selected at random, all possible pairs are equally likely. This means:

$$P(Q) = \frac{\text{number of ways to choose a pair of red bulbs}}{\text{number of ways to choose any pair of bulbs}}$$

$$P(Q) = \frac{{}_3C_2}{{}_{12}C_2} = \frac{\frac{3!}{2!1!}}{\frac{12!}{10!2!}} = \frac{3}{66} = \boxed{\frac{1}{22}}$$

b) The number of ways of picking one yellow and one red bulb is:

$$P\left(R\right)=\frac{{}_{9}C_{1}\bullet{}_{3}C_{1}}{{}_{12}C_{2}}=\frac{\frac{9!}{8!1!}\cdot\frac{3!}{2!1!}}{\frac{12!}{10!2!}}=\frac{9\cdot3}{66}$$

$$=\frac{27}{66}=\boxed{\frac{9}{22}}$$