Algebra 2 Probability

Algebra 2 Probability State Standards

1.0 Students know the definition of the notion of independent events and can use the rules for addition, multiplication, and complementation to solve for probabilities of particular events in finite sample spaces.

2.0 Students know the definition of conditional probability and use it to solve for probabilities in finite sample spaces.

Events are **independent events** if the occurrence of one event does not affect the probability of the other event.

For example, if a coin is flipped twice, whether the coin lands on heads or tails on the first flip, does not affect whether the coin lands on heads or tails on the second flip. This is also true for rolling a fair die. Whether a die lands on 1, 2, 3, 4, 5, or 6, on the first roll, does not affect the outcome of the second roll.

Probability of Independent Events

If $\boldsymbol{A}$ and $\boldsymbol{B}$ are independent events, then

$\mathrm{P}(\boldsymbol{A} \text{ and } \boldsymbol{B}) = \mathrm{P}(\boldsymbol{A}) \cdot \mathrm{P}(\boldsymbol{B})$

In other words, find the probability of each independent event and **multiply** the results together.

If you take a marble out of a bag, look at it's color and **place it back in the bag**, then the next selection of a marble will be an **independent event**. The same is true for taking a card from a deck of cards, looking at it and placing it back in the deck.

Ex 1) A bag contains 3 green marbles, 3 blue marbles, and 2 red marbles. If you put each marble back in the bag after looking at it, what is the probability of picking a red marble, a green marble, and a red marble?

P(red, green, red) = P(red) · P(green) · P(red)

$P(\text{red}) = \frac{2}{8} = \frac{1}{4}$

$P(\text{green}) = \frac{3}{8}$

$P(\text{red, green, red}) = \frac{1}{4} \cdot \frac{3}{8} \cdot \frac{1}{4} = \frac{3}{128}$

Events are **dependent events** if the occurrence of one event affects the probability of another event.

If you take a marble out of a bag and **don't place it back in the bag**, this affects the probability of the next selection **depending** on the outcome of the first selection.
The same is true for taking a card from a deck of cards, and **not returning it to the deck**.

To find the probability of dependent events, you can use **conditional probability P(*B* | *A*)**, the probability of event ***B***, given that event ***A*** has occurred.

Probability of Dependent Events
If *A* and *B* are dependent events, then
P(A and B) = P(A) · P(B | A), where P(*B* | *A*) is the probability of *B*, given that *A* has occurred.

Ex 2) A bag contains 5 red marbles and 4 blue marbles. What is the probability of randomly selecting two red marbles is you do not put the marbles back in the bag after selecting them?

1st Selection: $P(\text{red}) = \frac{5}{9}$

2nd Selection $P(\text{red then red}) = \frac{4}{8}$

Note: One less red marble = 4; one less marble to choose from = 8

$P(\text{both red}) = P(\text{red}) \cdot P(\text{red following red})$

$= \frac{5}{9} \cdot \frac{\cancel{4}^{1}}{\cancel{8}_{2}} = \frac{5}{18} = 0.278$

Ex 3) From a standard deck of 52 playing cards, what is the probability of randomly selecting a 2, followed by a 7, and then a 2, if the cards are **not** returned to the deck after being selected?

$P(2, 7, 2) = P(2) \cdot P(7 \mid 2) \cdot P(2 \mid 7 \mid 2)$

$$P(2,7,2) = \frac{4}{52} \cdot \frac{4}{51} \cdot \frac{3}{50} = \frac{48}{132,600}$$

$$= \frac{2}{5,525} \doteq 0.0004$$

When two events cannot happen at the same time, the events are called **mutually exclusive events**. If *A* and *B* are mutually exclusive events, then P(*A* and *B*) = 0.

Events that are not mutually exclusive are called **inclusive events** or **non-mutually exclusive events**.

Probability of (*A* or *B*)

If *A* and *B* are **mutually exclusive** events then,

P(A or B) = P(A) + P(B)

If *A* and *B* are **inclusive** events then,

P(A or B) = P(A) + P(B) - P(A and B).

Ex 4) A bag contains 3 dimes, 6 nickels, and 4 pennies. If one coin is selected at random, what is the probability that the coin is a penny **or** a nickel?

The coin can't be a penny **and** a nickel at the same time, so these are mutually exclusive events. We find the probability by calculating the sum of the individual probabilities.

P(penny or nickel) = P(penny) + P(nickel)

$$P(\text{penny or nickel}) = \frac{4}{13} + \frac{6}{13} = \frac{10}{13} = 0.769$$

Ex 5) A card is chosen at random from a standard deck of 52 playing cards. What is the probability that it is red **or** a face card (King, Queen, or Jack)?

Because some face cards are red (6 are red to be precise), these events are **not mutually exclusive** and are therefore, **inclusive**.

When we are finding the probability of one **or** the other of two **inclusive events**, *A* and *B*, we subtract the probability of both occurring from the sum of the individual probabilities.

P(*A* or *B*) = P(*A*) + P(*B*) - P(*A* and *B*)

P(red or face card) = P(red) + P(face card) - P(red face cards)

P(red or face card) $= \frac{36}{52} + \frac{12}{52} - \frac{6}{52}$

$= \frac{32}{52} = \frac{8}{13} = 0.615$

Ex 6) There are 6 women and 7 men on a committee. A subcommittee of 5 members is being randomly selected from the original committee. What is the probability that **at least** 3 women will be on this subcommittee?

The phrase **at least** means that this subcommittee can have 3, 4, **or** 5 women.

P(at least 3 women) = P(3 women) + P(4 women) + P(5 women)

P(at least 3 women) =

3 women 2 men — 4 women 1 man — 5 women 0 men

$$= \frac{{}_6C_3 \cdot {}_7C_2}{{}_{13}C_5} + \frac{{}_6C_4 \cdot {}_7C_1}{{}_{13}C_5} + \frac{{}_6C_5 \cdot {}_7C_0}{{}_{13}C_5}$$

$$= \frac{20 \cdot 21}{1287} + \frac{15 \cdot 7}{1287} + \frac{6 \cdot 1}{1287}$$

$$= \frac{420}{1287} + \frac{105}{1287} + \frac{6}{1287} = \frac{531}{1287} \begin{matrix} \div 9 \\ \div 9 \end{matrix}$$

$$= \frac{59}{143} = 0.413$$

The sum of all probabilities in a sample space is 1. The **complement** of an event *A* is the set of all outcomes in a sample space that are **not** in *A*.

Complement Probability
The complement of an event is the probability of that event **not** happening.

Complement
The probability of the complement of event *A* is
P(not *A*) = 1 - P(*A*)

Ex 7) Using a fair six sided die, what is the probability of **not** rolling a 2?

$$P(\text{not } 2) = 1 - P(2)$$
$$= 1 - \frac{1}{6}$$
$$= \boxed{\frac{5}{6} = 0.833}$$

Ex 8) On a certain day the chance of rain is 40% in Chicago and 20% in Los Angeles. Assume that the chance of rain in the two cities is independent. What is the probability that it will **not** rain in either city that day?

P(not rain in Chicago) · P(not rain in Los Angeles)

$$\begin{aligned} P(\text{not rain in Chicago}) &= 1 - P(\text{rain in Chicago}) \\ &= 1 - 0.40 \\ &= 0.60 \end{aligned}$$

$$\begin{aligned} P(\text{not rain in LA}) &= 1 - P(\text{rain in LA}) \\ &= 1 - 0.20 \\ &= 0.80 \end{aligned}$$

P(not rain in Chicago) · P(not rain in Los Angeles)

$0.6 \cdot 0.8 = 0.48 = 48\%$

Ex 9) A card is chosen at random from a standard deck of playing cards. Find the probability of drawing a Jack **or** a red card.

Because some red cards are Jacks (2 Jacks are red to be precise), these events are **not mutually exclusive** and are therefore, **inclusive**.

P(*A* or *B*) = P(*A*) + P(*B*) - P(*A* and *B*)

P(red or Jack) = P(red) + P(Jack) - P(red AND a Jack)

$$= \frac{26}{52} + \frac{4}{52} - \frac{2}{52}$$

$$= \frac{28}{52} = \frac{7}{13}$$