

A **matrix** is a rectangular array of numbers enclosed by square brackets (plural of matrix is matrices).

The numbers in a matrix are called the **elements** (or members) of the matrix. The number of **rows** (horizontal) and **columns** (vertical) determine the **dimensions** of the matrix.

A matrix with  $m$  rows and  $n$  columns is called a matrix whose dimensions are  $m \times n$  (rows before columns: RC Cola...).

We use capital letters for the name of a matrix and subscripts for the dimensions of the matrix.

A matrix with 2 rows and 3 columns may be written as:

$$A_{2 \times 3}$$

Here are some examples of matrices:

$$\begin{bmatrix} 3 & -10 & \frac{1}{2} \end{bmatrix} \quad \begin{bmatrix} 6 \\ 2 \end{bmatrix} \quad \begin{bmatrix} 1 & -1 \\ 0 & 5 \end{bmatrix}$$

$A_{1 \times 3} \qquad B_{2 \times 1} \qquad C_{2 \times 2}$

$$\begin{bmatrix} 9 & 1 & 0 \\ 0 & 3 & 2 \end{bmatrix}$$

$D_{2 \times 3}$

What are the dimensions of the matrices listed above?

A **square matrix** is a matrix that has the same number of rows and columns.

A **zero matrix** is a matrix where all the elements are zeros.

Ex 1) Write a zero matrix with 2 rows and 4 columns. Name this matrix E.

$$E_{2 \times 4} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Two matrices are equal if and only if ( iff ):

- 1) they have the same dimensions, and
- 2) the elements in all corresponding positions are equal.

For example:

$$\begin{bmatrix} 5 & 7 & \pi \\ \frac{7}{8} & \sqrt{2} & -10 \end{bmatrix} = \begin{bmatrix} 5 & 7 & \pi \\ \frac{7}{8} & \sqrt{2} & -10 \end{bmatrix}$$

Ex 2) Find the value of each variable (solve for x and y).

$$\begin{bmatrix} x+5 & -1 \\ 4 & 6 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 4 & 3y \end{bmatrix}$$

Since the matrices are equal, the elements in corresponding positions must be equal, therefore, we can solve as follows:

$$\textcircled{\text{I}} \quad x+5=2$$

$\quad \quad \quad -5 \quad -5$

$$\textcircled{x = -3}$$

$$\textcircled{\text{II}} \quad \frac{3y}{3} = \frac{6}{3}$$
$$\textcircled{y = 2}$$

An  $m \times n$  matrix, where  $m$  and  $n$  are positive integers, is an array in for form that follows.

(We can also refer to the elements in a matrix by their row and column position using subscripts as shown below.)

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \dots & a_{3n} \\ \dots & \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mn} \end{bmatrix}$$