16-2 Addition and Scalar Multiplication Page 770

Objective: To find sums and differences of matrices and products of a scalar and a matrix.

The sum of matrices having the same dimensions is the matrix whose elements are the sums of the corresponding elements of the matrices being added.

Strategy: To add or subtract matrices
1) They must have the same dimensions
2) Add or subtract the corresponding elements (elements in the same position in the matrices)

Ex. 1) Simplify: $\begin{bmatrix} 3 & -2 \\ 1 & 0 \\ -5 & 7 \end{bmatrix} + \begin{bmatrix} -6 & 4 \\ 0 & -9 \\ 3 & 0 \end{bmatrix}$

$$\begin{bmatrix} 3 & -2 \\ 1 & 0 \\ -5 & 7 \end{bmatrix} + \begin{bmatrix} -6 & 4 \\ 0 & -9 \\ 3 & 0 \end{bmatrix} = \begin{bmatrix} 3+-6 & -2+4 \\ 1+0 & 0+-9 \\ -5+3 & 7+0 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 2 \\ 1 & -9 \\ -2 & 7 \end{bmatrix}$$

Matrices with different dimensions do not have corresponding elements, therefore, addition of matrices with different dimensions is not defined.

For any $\boldsymbol{m \times n}$ matrices, $\boldsymbol{O_{m \times n}}$ is the identity for addition.

$$A_{2x2} + O_{2x2} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} a+0 & b+0 \\ c+0 & d+0 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

The additive inverse of matrix ***A*** is the matrix ***-A***. Each element in ***-A*** is the opposite of its corresponding element in ***A***. For example,

$$If\ A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix},\ then\ -A = \begin{bmatrix} -1 & -2 \\ -3 & -4 \end{bmatrix}$$

Adding two matrices that are *additive inverses* gives you the *additive identity*, which is a *zero matrix*.

Ex 2) Find ***A-B***

$$If\ A=\begin{bmatrix}1 & 6\\ -8 & 2\end{bmatrix}\ and\ B=\begin{bmatrix}4 & -2\\ 3 & 7\end{bmatrix}$$

$$A-B=\begin{bmatrix}1 & 6\\ -8 & 2\end{bmatrix}-\begin{bmatrix}4 & -2\\ 3 & 7\end{bmatrix}$$

Remember that A - B = A + (-B)

$$=\begin{bmatrix}1 & 6\\ -8 & 2\end{bmatrix}+\begin{bmatrix}-4 & 2\\ -3 & -7\end{bmatrix}=\begin{bmatrix}-3 & 8\\ -11 & -5\end{bmatrix}$$

Ex 3) $Let\ A=\begin{bmatrix}2 & 0\\ 5 & -7\end{bmatrix}$ Find $3A$

$$3A=3\begin{bmatrix}2 & 0\\ 5 & -7\end{bmatrix}$$

$$=\begin{bmatrix}3\cdot 2 & 3\cdot 0\\ 3\cdot 5 & 3\cdot -7\end{bmatrix}=\begin{bmatrix}6 & 0\\ 15 & -21\end{bmatrix}$$

An equation where a variable represents a matrix is called a *matrix equation*. Matrix equations can be solved using *matrix addition* and *scalar multiplication*.

Ex 4) Solve the matrix for X:

$$2X + 2\begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix} = 8\begin{bmatrix} \frac{1}{2} & 0 \\ -1 & \frac{1}{4} \end{bmatrix}$$

$$2X + \begin{bmatrix} 2 & -4 \\ 0 & 6 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ -8 & 2 \end{bmatrix}$$

$$2X = \begin{bmatrix} 4 & 0 \\ -8 & 2 \end{bmatrix} - \begin{bmatrix} 2 & -4 \\ 0 & 6 \end{bmatrix}$$

$$2X = \begin{bmatrix} 4 & 0 \\ -8 & 2 \end{bmatrix} + \begin{bmatrix} -2 & 4 \\ 0 & -6 \end{bmatrix}$$

$$2X = \begin{bmatrix} 2 & 4 \\ -8 & -4 \end{bmatrix}$$

$$X = \frac{1}{2}\begin{bmatrix} 2 & 4 \\ -8 & -4 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 & 2 \\ -4 & -2 \end{bmatrix}$$