Every square matrix has a real number associated with it called its **determinant**. The determinant of matrix A is denoted by $|A|$ or by **det A**.
The number of elements in any row or column (since the number of rows and columns is the same) is called the **order** of the determinant.

The determinant of matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is denoted by $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$
The det A is the same as |A|.

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - cb$$

Ex 1) Evaluate $\begin{vmatrix} 7 & -3 \\ -4 & -8 \end{vmatrix}$ $= 7 \cdot (-8) - (-4) \cdot (-3)$

$= -56 - 12$

$= -68$

The determinant of a 3 x 3 matrix B, **det B** is defined as follows:

$$\det B = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = aei + bfg + cdh - gec - hfa - idb$$

You can remember this process using the following steps:

1) Copy the first 2 columns of the matrix after the matrix:

$$\begin{vmatrix} a & b & c & a & b \\ d & e & f & d & e \\ g & h & i & g & h \end{vmatrix}$$

2) Multiply all the diagonals from left to right **going down** that **contain 3 elements** and **add** these results together.

$$\begin{vmatrix} a & b & c & a & b \\ d & e & f & d & e \\ g & h & i & g & h \end{vmatrix} = \boldsymbol{aei + bfg + cdh}$$

3) Multiply all the diagonals from left to right **going up** that **contain 3 elements** and **subtract** these results from part (2).

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} \begin{matrix} a & b \\ d & e \\ g & h \end{matrix} \quad \textbf{- gec - hfa - idb}$$

Important! This only works for 3 x 3 determinants!

Ex 2) Evaluate det $A = \begin{vmatrix} -1 & 0 & 1 \\ -5 & 1 & -1 \\ 4 & 8 & 1 \end{vmatrix}$

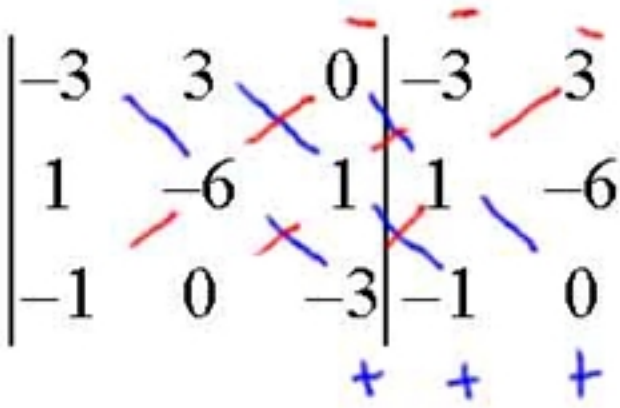
$$\begin{vmatrix} -1 & 0 & 1 \\ -5 & 1 & -1 \\ 4 & 8 & 1 \end{vmatrix} \begin{matrix} -1 & 0 \\ -5 & 1 \\ 4 & 8 \end{matrix} = (-1)(1)(1) + 0(-1)(4) + (1)(-5)(8) - (4)(1)(1) - (8)(-1)(-1) - (1)(-5)(0)$$

$= -1 + 0 - 40 - 4 - 8 - 0$

$= -41 - 12$

$= \boxed{-53}$

Ex 2) Evaluate det B $\begin{vmatrix} -3 & 3 & 0 \\ 1 & -6 & 1 \\ -1 & 0 & -3 \end{vmatrix}$



$= (-3)(-6)(-3) + (3)(1)(-1) + (0)(1)(0) - (-1)(-6)(0) - (0)(1)(-3) - (-3)(1)(3)$

$= -54 - 3 + 0 - 0 - 0 + 9$

$= -57 + 9$

$= -48$