The **minor of an element** in a determinant **is the determinant** formed by "deleting" or "hiding" the row and column that contains the element. (The minor matrix of a *3x3* matrix will be a *2x2* matrix.)

To Calculate the Determinant using Expansion by Minors

1) Select a row or column to use.
2) Multiply each element in this row or column by the determinant of its minor matrix.
3) Add the row and column numbers for each element. If the sum is odd, multiply the product obtained in (2) by -1. Or use the matrix of alternating signs listed below.
4) Add the products to find the value of the determinant.

$$\begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = +a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

Ex 1) Evaluate the determinant of this matrix using expansion by minors. Use the first row to find the minor matrices.

$$\begin{vmatrix} -1 & 0 & 1 \\ -5 & 1 & -1 \\ 4 & 8 & 1 \end{vmatrix}$$

$$\begin{vmatrix} -1 & 0 & 1 \\ -5 & 1 & -1 \\ 4 & 8 & 1 \end{vmatrix} = +(-1)\begin{vmatrix} 1 & -1 \\ 8 & 1 \end{vmatrix}$$

$$\begin{vmatrix} -1 & 0 & 1 \\ -5 & 1 & -1 \\ 4 & 8 & 1 \end{vmatrix} = +(-1)\begin{vmatrix} 1 & -1 \\ 8 & 1 \end{vmatrix} - 0\begin{vmatrix} -5 & -1 \\ 4 & 1 \end{vmatrix}$$

$$\begin{vmatrix} -1 & 0 & 1 \\ -5 & 1 & -1 \\ 4 & 8 & 1 \end{vmatrix} = +(-1)\begin{vmatrix} 1 & -1 \\ 8 & 1 \end{vmatrix} - 0\begin{vmatrix} -5 & -1 \\ 4 & 1 \end{vmatrix} + 1\begin{vmatrix} -5 & 1 \\ 4 & 8 \end{vmatrix}$$

$= -1(1 - -8) - 0(-5 - -4) + 1(-40 - 4)$

$= -1(9) - 0(-1) + 1(-44)$

$= -9 - 0 - 44$

$= -53$

Ex 2) Evaluate the determinant of this matrix using expansion by minors. Use the third column to find the minor matrices.

$$\begin{vmatrix} -3 & 3 & 0 \\ 1 & -6 & 1 \\ -1 & 0 & -3 \end{vmatrix}$$

$$\begin{vmatrix} -3 & 3 & 0 \\ 1 & -6 & 1 \\ -1 & 0 & -3 \end{vmatrix} = +0\begin{vmatrix} 1 & -6 \\ -1 & 0 \end{vmatrix}$$

$$\begin{vmatrix} -3 & 3 & 0 \\ 1 & -6 & 1 \\ -1 & 0 & -3 \end{vmatrix} = +0\begin{vmatrix} 1 & -6 \\ -1 & 0 \end{vmatrix} - 1\begin{vmatrix} -3 & 3 \\ -1 & 0 \end{vmatrix}$$

$$\begin{vmatrix} -3 & 3 & 0 \\ 1 & -6 & 1 \\ -1 & 0 & -3 \end{vmatrix} = +0\begin{vmatrix} 1 & -6 \\ -1 & 0 \end{vmatrix} - 1\begin{vmatrix} -3 & 3 \\ -1 & 0 \end{vmatrix} + -3\begin{vmatrix} -3 & 3 \\ 1 & -6 \end{vmatrix}$$

$$= 0(0-6) - 1(0 - {}^{-}3) - 3(18-3)$$

$$= 0(-6) - 1(+3) - 3(15)$$

$$= 0 - 3 - 45$$

$$= \boxed{-48}$$