Now that we know how to calculate the determinant of a square matrix, we can use **Cramer's Rule** to solve systems equations.

Let's start with 2 equations and 2 variables...

$$ax + by = c$$

$$dx + ey = f$$

We need to calculate the determinants for 3 - two by two matrices. One will represent a denominator, a second will represent the numerator of x in our answer and the third will represent the numerator of y in our answer.

$$D_d = \begin{vmatrix} a & b \\ d & e \end{vmatrix}$$

This determinant will give us the denominator.

$$D_x = \begin{vmatrix} c & b \\ f & e \end{vmatrix}$$

This determinant will give us the numerator of x.

$$D_y = \begin{vmatrix} a & c \\ d & f \end{vmatrix}$$

This determinant will give us the numerator of y.

Putting these determinants together gives us our answer.

$$x = \frac{D_x}{D_d} = \frac{\begin{vmatrix} c & b \\ f & e \end{vmatrix}}{\begin{vmatrix} a & b \\ d & e \end{vmatrix}}$$

$$y = \frac{D_y}{D_d} = \frac{\begin{vmatrix} a & c \\ d & f \end{vmatrix}}{\begin{vmatrix} a & b \\ d & e \end{vmatrix}}$$

Ex 1) Solve the system of equations.

$$2x+5y=7$$

$$4x-2y=-3$$

$$D_d=\begin{vmatrix}2 & 5\\ 4 & -2\end{vmatrix} = (2)(-2)-(4)(5)$$

$$= -4-20 = \boxed{-24}$$

$$D_x=\begin{vmatrix}7 & 5\\ -3 & -2\end{vmatrix} = (7)(-2)-(-3)(5)$$

$$= -14+15 = \boxed{1}$$

$$D_y=\begin{vmatrix}2 & 7\\ 4 & -3\end{vmatrix} = 2(-3)-(4)(7)$$

$$-6-28 = \boxed{-34}$$

$$x = \frac{D_x}{D_d} = \frac{-1}{24}$$

$$y = \frac{D_y}{D_d} = \frac{-34}{-24} = \frac{17}{12}$$

$\left(\frac{-1}{24}, \frac{17}{12}\right)$

Consistent
Independent

Ex 2) Solve the system of equations.

$$2x - y = 6$$

$$3x + 5y = 22$$

$$D_d = \begin{vmatrix} 2 & -1 \\ 3 & 5 \end{vmatrix} = (2)(5) - (3)(-1) = 10 + 3 = \boxed{13}$$

$$D_x = \begin{vmatrix} 6 & -1 \\ 22 & 5 \end{vmatrix} = (6)(5) - (22)(-1) = 30 + 22 = \boxed{52}$$

$$D_y = \begin{vmatrix} 2 & 6 \\ 3 & 22 \end{vmatrix} = (2)(22) - (3)(6) = 44 - 18 = \boxed{26}$$

$$x = \frac{D_x}{D_d} = \frac{52}{13} = 4$$

$$y = \frac{D_y}{D_d} = \frac{26}{13} = 2$$

(4, 2)
Consistent
Independent

How do you know when the system is **inconsistent** or **dependent?**

If $D_d = 0$ AND $D_y \neq 0$ the equations are inconsistent and their graphs are parallel.

If $D_d = 0$ AND $D_y = 0$ the equations are consistent and dependent (graphs are the same line).