

Algebra 2 Standard: 2.0 Students solve systems of linear equations and inequalities (in two or three variables) by substitution, with graphs, or with matrices.

We will be learning several methods for solving systems of Linear Equations:

- 1) Graphing
- 2) Substitution Method
- 3) Linear Combination (a.k.a. Elimination or Addition)

Other methods will come in later chapters.

In the graphing method we will be graphing both linear equations on the same plane. One of three possibilities will occur. The graphs will:

- 1) Intersect at one point (one solution).
- 2) The graphs will be parallel (no solutions).
- 3) The graphs will coincide or will be the same line. (There are an infinite number of ordered pairs in our solution, namely all the ordered pairs on both lines.)

Whatever method we use, there are some vocabulary terms to describe our solutions. If there is a solution (#1, & #3 above), we say that the system is ***consistent***. We need to distinguish between #1 & #3 above, so if the solution is one point (#1) we say that the system is also ***independent***. If there are an infinite number of solutions (#3), the system is ***dependent***. If there is no solution (#2 above) we say that the system is ***inconsistent***.

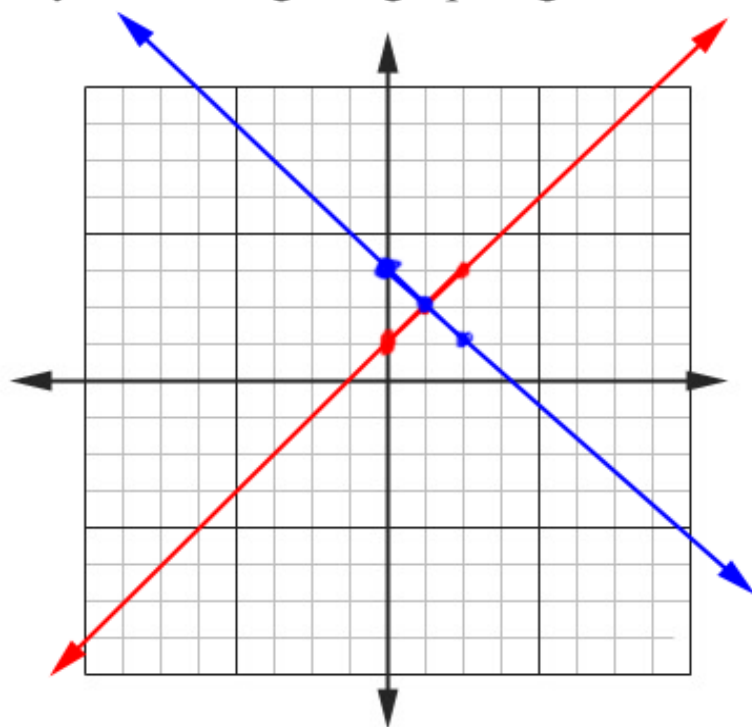
Ex 1) Solve the system using the graphing method.

$$y = x + 1$$

$$y = -x + 3$$

Solution:  
(1, 2)

consistent  
independent

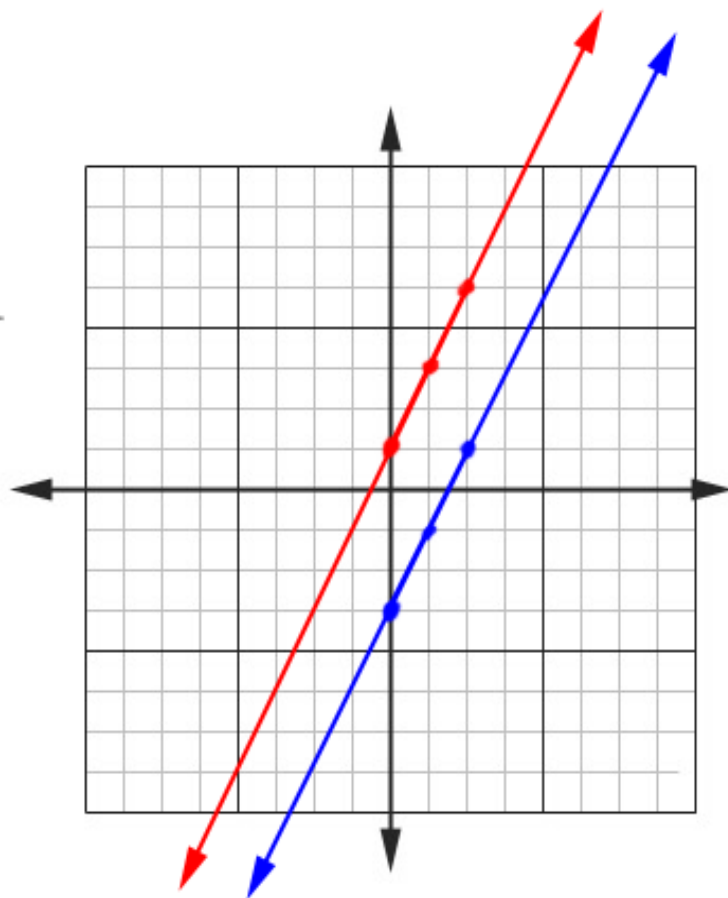


Ex 2) Solve the system using the graphing method.

$$y = 2x + 1$$

$$y = 2x - 3$$

No Solution  
Inconsistent



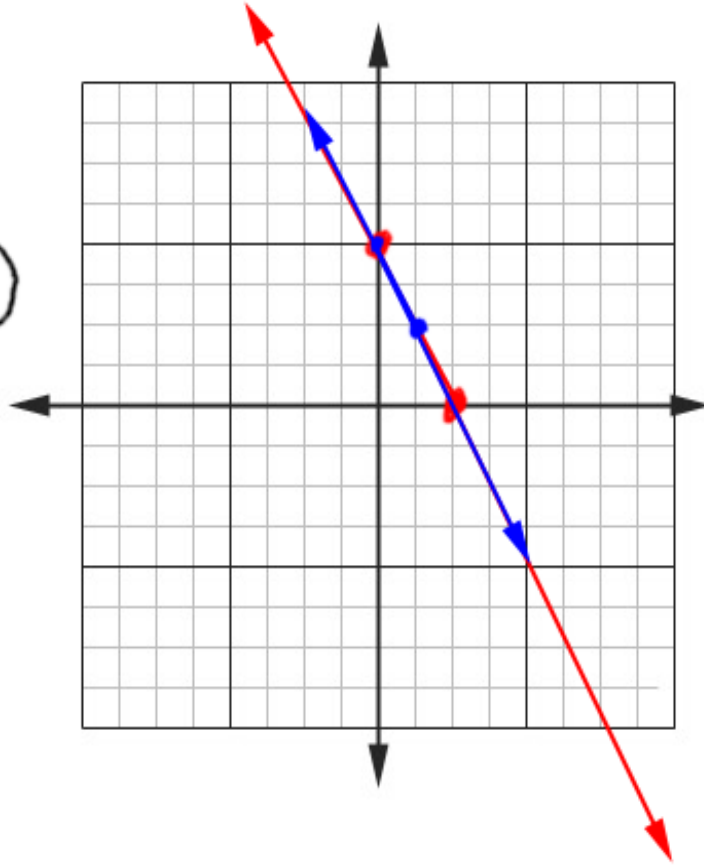
Ex 3) Solve the system using the graphing method.

$$4x + 2y = 8$$

$$y = -2x + 4$$

$$y = -2x + 4$$

consistent  
dependent



The substitution method is useful if one of the variables in the system has a coefficient of 1 (one).

Ex 4) Solve the system using the substitution method.

$$x + y = 5$$

$$x = y + 1$$

We will replace  $x$  in the first equation with  $(y + 1)$  from the second equation since  $x$  "is the same as"  $y + 1$ .

$$\begin{aligned} \textcircled{\text{I}} \quad (y + 1) + y &= 5 \\ 2y + 1 &= 5 \\ 2y &= 4 \\ y &= 2 \end{aligned}$$

(Half our ordered pair)

$$\begin{aligned} \textcircled{\text{II}} \quad x &= y + 1 \\ x &= 2 + 1 \\ x &= 3 \end{aligned}$$

Soln:  $(3, 2)$   
consistent  
independent

Linear combination is the process of adding the two equations together for the purpose of eliminating one of the variables. Frequently we will have to multiply one of the equations (or even both of the equations) by a number in order to get coefficients of one of the variables that are opposites (and will be eliminated when adding).

Ex 5) Solve the system using Linear Combination

$$3x + 3y = 15$$

$$2x + 6y = 22$$

$$\textcircled{\text{I}} \quad \begin{array}{r} -2(3x + 3y = 15) \rightarrow -6x - 6y = -30 \\ 2x + 6y = 22 \\ \hline -4x = -8 \end{array}$$

$$x = 2$$

(half our ordered pair)

$$\textcircled{\text{II}} \quad 3(2) + 3y = 15$$

$$6 + 3y = 15$$

$$3y = 9$$

$$y = 3$$

Solution:  $(2, 3)$  consistent, independent

Ex 6) Solve the system using Linear Combination

$$5x + 4y = 11$$

$$3x - 5y = -23$$

$$\textcircled{\text{I}} \begin{array}{r} -3(5x + 4y = 11) \rightarrow -15x - 12y = -33 \\ + 5(3x - 5y = -23) \rightarrow +15x - 25y = -115 \\ \hline \end{array}$$

$$\begin{array}{r} -37y = -148 \\ \hline -37 \quad -37 \end{array}$$

$$y = 4$$

$$\begin{array}{l} \textcircled{\text{II}} \quad 5x + 4(4) = 11 \\ \quad \quad 5x + 16 = 11 \\ \quad \quad 5x = -5 \\ \quad \quad x = -1 \end{array}$$

Solution  $(-1, 4)$  consistent, independent