Factoring Polynomials

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Algebra 2 Standards:

3.0 Students are adept at operations on polynomials, including long division.

4.0 Students factor polynomials representing the difference of squares, perfect square trinomials, and the sum and difference of two cubes.
Factor

• When you see the word “Factor” in a problem, first understand that you have to factor *completely*.

• This is called “Prime Factorization”
  – Where each factor is prime, and therefore, cannot be factored any further.

• When factoring monomials or constants (numbers), a factor tree will help find the prime factors.
Factoring Polynomials

• When factoring Polynomials there are five steps to follow (in the order that is given in the next slide) to make sure that you have factored the polynomial into its prime factors.

• You will lose points if the polynomial is not completely factored into its prime factors.

• By the way, some polynomials (very few) will not be able to be factored.
Steps To Factoring Polynomials

I. Factor out the GCF of the polynomial.

II. Factor using the 5 Special Formulas.
   (slides 9 – 20)

III. Use the FOIL process “backwards” (LIOF) for any polynomials in the form:
   \[1x^2 + bx + c\]

IV. Use the “Trial & Error” process for any polynomials in the form:
   \[ax^2 + bx + c\]

V. If there are four terms, use the “Factoring By Grouping” method.
I. Finding The GCF

- Factor each term in the polynomial into its prime factorization (prime factors).
- Calculate the Greatest Common Factor.
- Use the Distributive Property “backwards” to factor out the GCF.
  \[ ab + ac = a(b + c) \]
- To make sure that the GCF is factored correctly, use the Distributive Property “forwards” to check your answer.
  \[ a(b + c) = ab + ac \]
GCF Example 1a

Ex 1a) Factor: $2x^4 - 4x^3 + 8x^2$

The Prime Factorization of each term is:
$2 \cdot x^4; \ -1 \cdot 2^2 \cdot x^3; \ and \ 2^3 \cdot x^2$

The GCF is: $2x^2$

Using the Distributive Property “backwards”
Divide each term of the polynomial by the GCF to get:
$2x^2 \ (x^2 - 2x + 4)$
GCF Example 1b

Ex 1b) Factor: $10ab^3 - 15a^2b^2$

The Prime Factorization of each term is:
$2 \cdot 5ab^3; \text{ and } -1 \cdot 3 \cdot 5a^2b^2$

What is the GCF?
$5ab^2$

Divide each term of the polynomial by the GCF to get:
$5ab^2 (2b - 3a)$
II. Special Formulas

The first two special formulas to memorize:

A. and B. **Perfect Square Trinomials**

\[ a^2 + 2ab + b^2 = (a + b)^2 \]
\[ a^2 - 2ab + b^2 = (a - b)^2 \]

To use this formula, the polynomial must meet 5 criteria:

1) The polynomial is a trinomial
2) The last term is positive
3) The first term is a perfect square
4) The last term is a perfect square
5) The product of the square roots of the first and last term doubled must equal the middle term of the polynomial
Perfect Square Trinomial Ex 2a

Ex 2a) Factor: $z^2 + 6z + 9$
It is a trinomial, the last term is positive,
the first and last terms are perfect squares…
Let’s look at step #5:
Does $z$ times 3 doubled equal $6z$?
If so, then this factored as:
$(z + 3)^2$
Perfect Square Trinomial Ex 2b

Ex 2b) Factor: $4s^2 - 4st + t^2$

It is a trinomial, the last term is positive, the first and last terms are perfect squares…

Let’s look at step #5:
Does $2s$ times $t$ doubled equal $4st$?
It needs to equal $-4st$, so we factor it as:
$(2s - t)^2$
II. Special Formulas

The third special formula to memorize:

C. The Difference of two Perfect Squares

\[ a^2 - b^2 = (a + b) (a - b) \]

To use this formula, the polynomial must meet 4 criteria:
1) The polynomial is a binomial
2) The two terms are subtracted
3) The first term is a perfect square
4) The last term is a perfect square

Then factor the polynomial into its two conjugates:

\[ (a + b) (a - b) \]
Ex 2c) Factor: $25x^2 - 16a^2$

It is a binomial, the terms are subtracted, the first and last terms are perfect squares…

What are the conjugates using the square roots of the first and last terms?

$25x^2 - 16a^2 = (5x + 4a)(5x - 4a)$
Example 3

Ex 3) Factor: 3x^5 - 48x

1st Is there a GCF? If so, what is it?
Yes, it’s 3x. So we have 3x(x^4 - 16)

2nd, Is the polynomial (x^4 - 16) one of the special formulas? If so, which one?
Yes it’s the difference of 2 perfect squares…

We have: 3x(x^2 - 4)(x^2 + 4)

Are any of these factors special formulas?
Yes, (x^2 - 4) is the difference of 2 perfect squares, so 3x (x-2)(x+2) (x^2 + 4) is our prime factorization
Homework

• To practice what we have learned so far…
II Special Formulas

• The fourth and fifth special formulas:

• D. The Difference of Two Perfect Cubes
  \[ a^3 - b^3 = (a-b)(a^2 + ab + b^2) \]

• E. The Sum of Two Perfect Cubes
  \[ a^3 + b^3 = (a+b)(a^2 - ab + b^2) \]

To use these formulas, the polynomials must meet 4 criteria:

1) The polynomial is a binomial
2) The first term is a perfect cube
3) The last term is a perfect cube
4) The two terms are subtracted or added depending which formula you are using.
Sum and Difference of Cubes

• Once you recognize that the polynomial is the sum or difference of perfect cubes:
  - Your factored answer consists of a binomial times a trinomial.
    • To find the binomial, take the cube roots of the two terms, place them in the factored binomial and use the sign of the original problem.
    • \( a^3 - b^3 = (a-b)(\text{trinomial}) \)
    • \( a^3 + b^3 = (a+b) (\text{trinomial}) \)
Sum and Difference of Cubes

- To find the trinomial, refer to the factored binomial
  
  \[ a^3 - b^3 = (a-b)(\text{trinomial}) \]
  
  \[ a^3 + b^3 = (a+b)(\text{trinomial}) \]

- Take the first term and square it

- Multiply the two terms and take the opposite

- Take the last term, including its sign and square it.

\[ a^3 - b^3 = (a-b)(a^2 + ab + b^2) \]

\[ a^3 + b^3 = (a+b)(a^2 - ab + b^2) \]
Example 4a Page 184

• Factor: \( y^3 - 1 \)
  - Take the cube root of each term for the binomial: \( (y-1) \)
    • To find the trinomial, refer to the binomial
    • Take the first term and square it
    • Multiply the two terms and take the opposite
    • Take the last term, including its sign and square it.

\[ (y-1)(y^2 + y + 1) \]
Example 4b Page 184

• Factor: $8u^3 + v^3$
  
  – Take the cube root of each term for the binomial: $(2u+v)$
    
    • To find the trinomial, refer to the binomial
    • Take the first term and square it
    • Multiply the two terms and take the opposite
    • Take the last term, including its sign and square it.

$$(2u+v)(4u^2 - 2uv + v^2)$$
Homework

• To practice what we have learned so far...
III Foil Backwards (LIOF)

- There is no guess work involved in factoring polynomials in the form $1x^2 + bx + c$
  
  • These polynomials usually factor into two binomials
  • First, determine the signs of the binomials as follows:
    
    $1x^2 + bx + c = (x + ?)(x + ?)$ Use factors of “c” that add to the middle term
    
    $1x^2 - bx + c = (x - ?)(x + ?)$ Use factors of “c” that subtract to the middle term
    
    $1x^2 - bx - c = (x + ?)(x - ?)$ add to the middle term
    
    $1x^2 + bx - c = (x - ?)(x + ?)$ subtract to the middle term

- List all the possible ways of factoring “c”
Factor: $x^2 + 2x - 15$

• From the previous slide we know that we can begin to factor this as:

$$(x + ?)(x - ?)$$

• Next, we need to list all the possible ways of factoring the constant, 15:

1 • 15 and 3 • 5

• Which factors subtract to give us $+ 2x$?

• Since the middle term is positive, we place the larger of the two factors with the plus sign, and the smaller of the two factors with the minus sign.

$$(x + 5)(x - 3)$$
Example 3 Page 189

Factor: $3 - 2z - z^2$
Rewrite this as: $-z^2 - 2z + 3$
Factor out a $-1$ to get: $-1(z^2 + 2z - 3)$
Now we can factor this as: $-1(z + ?)(z - ?)$
The only way to factor 3 is: $1 \cdot 3$
  - Which factors subtract to give us $+2z$?
  - Since the middle term is positive, we place the larger of the two factors with the plus sign, and the smaller of the two factors with the minus sign.

$-1(z + 3)(z - 1)$
Example

Factor: $x^2 - 20x + 36$

- We know that we can begin to factor this as:

$(x - ?)(x - ?)$

- Next, we need to list all the possible ways of factoring the constant, 36:

$1 \cdot 36, \ 2 \cdot 18, \ 3 \cdot 12, \ 4 \cdot 9, \ 6 \cdot 6$

- Which factors ADD to give us $20x$?

- (Don’t worry that it’s -20x, the signs are taken care of.) We can factor this as:

$(x - 2)(x - 18)$
IV Trial & Error

• Use the “Trial & Error” process for any polynomial in the form $ax^2 + bx + c$
• Polynomials in the form of $ax^2 + bx + c$ or $1x^2 + bx + c$ are called “quadratic polynomials”
• The $ax^2$ or the $1x^2$ term is called the “quadratic term”
• The $bx$ term is called the linear term
• “c” is called the constant.
Trial & Error 2

• Based upon the name, you can perceive that there is some guess work in this method.
  – There is no guess work to determine the signs of the factored binomials.

• First, determine the signs of the binomials as follows:

  \[ ax^2 + bx + c = (?x + ?)(?x + ?) \]
  \[ ax^2 - bx + c = (?x - ?)(?x - ?) \]
  \[ ax^2 - bx - c = (?x + ?)(?x - ?) \]
  \[ ax^2 + bx - c = (?x - ?)(?x + ?) \]
Trial & Error 3

– After determining the signs, we need to list all the possible ways of factoring “a” (the coefficient of $x^2$) and “c” (the constant).
– Next, we need to try multiplying all the possible combinations of the factors of “a” with all the possible factors of “c”
– This is best illustrated by example.
Example 2 Page 188

• Factor $15t^2 - 16t + 4$
  
  – From slide 25 we know that the signs are:
  
  $(\_ t - \_)(\_ t - \_)$

List all the possible ways to factor of 15 and 4

1 • 15
3 • 5
1 • 4
4 • 1
2 • 2

We have to consider (do the OI part of FOIL) all the combinations of the factors of 15 times the factors of 4 to see which ones will give us the middle term 16t
Example 2 Page 188 cont.

• Factor $15t^2 - 16t + 4$
  – Here are the 6 combinations and the middle terms:

  (1t - 1)(15t - 4)  \(-19t\)
  (1t - 4)(15t - 1)  \(-61t\)
  (1t - 2)(15t - 2)  \(-32t\)
  (3t - 1)(5t - 4)  \(-17t\)
  (3t - 4)(5t - 1)  \(-23t\)
  (3t - 2)(5t - 2)  \(-16t \quad !!!!!!\)

Our Factors are: $(3t - 2)(5t - 2)$
• **Factor 10t^2 + 11t - 6**

(1t - 1)(10t + 6) \(-10t + 6t = -4t\)

(1t - 6)(10t + 1) \(-60t + 1t = -59t\)

(1t - 2)(10t + 3) \(-20t + 3t = -17t\)

(1t - 3)(10t + 2) \(-30t + 2t = -28t\)

(2t - 1)(5t + 6) \(-5t + 12t = 7t\)

(2t - 6)(5t + 1) \(-30t + 2t = -28t\)

(2t - 2)(5t + 3) \(-10t + 6t = -4t\)

(2t - 3)(5t + 2) \(-15t + 4t = -11t\) We have the correct number 11, but the wrong sign.

So swap the signs in the parentheses to get the answer: \((2t + 3)(5t - 2)\)
Homework

• To practice what we have learned so far…
  
  factor.cpp
  
  factor.exe
  
  polyfact.bas
V Factoring By Grouping

- Factoring by grouping is used when you have four terms to factor.
- The steps are:
  - 1) Look for a GCF common to all four terms
  - 2) Look for three terms that make a special formula
  - 3) Look for two terms that make a special formula
  - 4) See if you can group the four terms into two groups of two. Each group needs to have a GCF.
    - When factored, each group needs to have another GCF
Example 5a Page 185

- Factor: 3xy - 4 - 6x + 2y
  - The 1st and 3rd terms have a GCF
  - The 2nd and 4th terms have a GCF
  - Rewrite this as:
    (3xy - 6x) + (2y - 4) What property is this?
  Factor out the GCF’s
  3x(y - 2) + 2(y - 2) Do these terms have a GCF?
  Yes, the (y-2). Factor this out of both terms:
  (y - 2) (3x + 2)
Example 5b Page 185

- Factor: $s^2 - 4t^2 - 4s + 4$
  - Do any three terms make a special formula?
  - Rewrite this as:
    $(s^2 - 4s + 4) - 4t^2$  (A perfect square trinomial)
    The first 3 terms factor as:
    $(s - 2)^2 - 4t^2$  We don’t have all factors yet...

Is there another special formula here?
Yes! The difference of 2 perfect squares.

$[(s - 2) - 2t] \cdot [(s - 2) + 2t]$ or
$(s - 2 - 2t) \cdot (s - 2 + 2t)$
Homework

• To practice what we have learned so far…