

A rational algebraic expression is a quotient of polynomials.

To simplify rational algebraic expressions:

- 1) Factor the numerator completely,
- 2) Factor the denominator completely,
- 3) Reduce any like FACTORS (not terms)

Ex 1) Simplify: $\frac{x^2 - 5x}{x^2 - 25}$

$$= \frac{x(x-5)}{(x+5)(x-5)}$$

$$= \frac{x(\cancel{x-5})}{(x+5)(\cancel{x-5})}$$

$$= \frac{x}{x+5}$$

Ex 2) Simplify:

$$(9x^2 + 6xy - 3y^2)(12x^2 - 12y^2)^{-1}$$

$$= \frac{9x^2 + 6xy - 3y^2}{12x^2 - 12y^2}$$

$$= \frac{3(3x^2 + 2xy - y^2)}{12(x^2 - y^2)}$$

$$= \frac{3(3x - y)(x + y)}{2 \cdot 2 \cdot 3(x + y)(x - y)}$$

$$= \frac{\cancel{3}(3x - y)\cancel{(x + y)}}{2 \cdot 2 \cdot \cancel{3}\cancel{(x + y)}(x - y)}$$

$$= \frac{3x - y}{4(x - y)}$$

A **rational function** is a function that is defined by a simplified rational expression in one variable.

Ex 3) Let $f(x) = \frac{2x^2 - 7x + 3}{x^3 + x^2 - 2x}$

3a) Find the domain of $f(x)$.

(Translation: What values of x are not allowed in the denominator?)

Remember we can't have zero in the denominator of a fraction.)

$$f(x) = \frac{(2x - 1)(x - 3)}{x(x - 1)(x + 2)}$$

Zero Prod. Prop.

$$x \neq 0 \quad x - 1 \neq 0 \quad x + 2 \neq 0$$

$$x \neq 0 \quad x \neq 1 \quad x \neq -2$$

Domain is \mathbb{R} except 0, 1, -2

3b) Find the zeros of f , if any:

Hint: set the variables of the numerator equal to zero to find the zeros of the functions.

$$2x - 1 = 0 \quad \text{or} \quad x - 3 = 0$$

$$2x = 1$$

$$x = 3$$

$$x = \frac{1}{2}$$

or