A rational algebraic expression is a quotient of polynomials.

To simplify rational algebraic expressions:
1) Factor the numerator completely,
2) Factor the denominator completely,
3) Reduce any like FACTORS (not terms)

Ex 1) Simplify: \[ \frac{x^2 - 5x}{x^2 + 25} \]

\[ = \frac{x(x - 5)}{(x + 5)(x - 5)} \]

\[ = \frac{x}{x + 5} \]
Ex 2) Simplify:

\[
(9x^2 + 6xy - 3y^2)(12x^2 - 12y^2)^{-1}
\]

= \[
\frac{9x^2 + 6xy - 3y^2}{12x^2 - 12y^2}
\]

= \[
\frac{3(3x^2 + 2xy - y^2)}{12(x^2 - y^2)}
\]

= \[
\frac{3(3x - y)(x + y)}{2 \cdot 2 \cdot 3 (x+y)(x-y)}
\]

= \[
\frac{3(3x - y)(x + y)}{2 \cdot 2 \cdot 3 (x+y)(x-y)}
\]

= \[
\frac{3x - y}{4(x-y)}
\]

A rational function is a function that is defined by a simplified rational expression in one variable.
Ex 3) Let \( f(x) = \frac{2x^2 - 7x + 3}{x^3 + x^2 - 2x} \)

3a) Find the domain of \( f(x) \).
(Translation: What values of \( x \) are not allowed in the denominator?
Remember we can't have zero in the denominator of a fraction.)

\[
f(x) = \frac{(2x - 1)(x - 3)}{x(x - 1)(x + 2)}
\]

Zero Prod. Prop.
\[
\begin{align*}
x \neq 0 & \quad x - 1 \neq 0 & \quad x + 2 \neq 0 \\
x \neq 0 & \quad x \neq 1 & \quad x \neq -2
\end{align*}
\]

Domain is \( \mathbb{R} \) except 0, 1, 2
3b) Find the zeros of \( f \), if any:

Hint: set the variables of the numerator equal to zero to find the zeros of the functions.

\[
2x - 1 = 0 \quad \text{or} \quad x - 3 = 0
\]

\[
2x = 1 \quad \Rightarrow \quad x = \frac{1}{2}
\]

\[
x = 3
\]