

A fraction is a **Complex Fraction** if its numerator or denominator or both contain one or more fractions or exponents that are negative.

For example:
$$\frac{2x - \frac{4x}{3y}}{\frac{5x^2 + 2x}{6y^2}} \quad \text{or} \quad \frac{a^{-1} + b^{-1}}{a^{-3} + b^{-3}}$$

Instead of thinking of these as hard "complex" problems (bad pun intended), think of these problems as 3 easier problems in one.

- 1) Simplify the numerator.
- 2) Simplify the denominator.
- 3) Divide.

Ex 1) Simplify:

$$\frac{1 + \frac{1}{x}}{1 - \frac{1}{x}}$$

$$\frac{1 + \frac{1}{x}}{1 - \frac{1}{x}} \rightarrow \frac{\frac{x}{x} + \frac{1}{x}}{\frac{x}{x} - \frac{1}{x}} \rightarrow \frac{\frac{x+1}{x}}{\frac{x-1}{x}}$$

$$= \frac{x+1}{x} \div \frac{x-1}{x}$$

$$= \frac{x+1}{x} \cdot \frac{x}{x-1} \rightarrow \frac{\cancel{x}(x+1)}{\cancel{x}(x-1)}$$

$$= \frac{x+1}{x-1}$$

Ex 2) Simplify: $\frac{1-3x^{-1}}{1-2x^{-1}-3x^{-2}}$

$$= \frac{1 - \frac{3}{x}}{1 - \frac{2}{x} - \frac{3}{x^2}}$$

$$= \frac{\frac{x}{x} - \frac{3}{x}}{\frac{x^2}{x^2} - \frac{2x}{x^2} - \frac{3}{x^2}} \rightarrow \frac{\frac{x-3}{x}}{\frac{x^2-2x-3}{x^2}}$$

$$= \frac{x-3}{x} \div \frac{x^2-2x-3}{x^2}$$

$$= \frac{x-3}{x} \cdot \frac{x^2}{x^2-2x-3}$$

$$= \frac{(x-3) x \cdot x}{x(x-3)(x+1)}$$

$$= \frac{\cancel{(x-3)} \cancel{x} \cdot x}{\cancel{x} \cancel{(x-3)}(x+1)}$$

$$= \frac{x}{x+1}$$