A **fractional equation** is an equation in which a variable occurs in the denominator.

To solve fractional equations:

1) Factor all denominators completely.
2) Determine the Domain. (Find the values of the variables that would make the denominator a zero. We must restrict or exclude these values from our solution set.)
3) Find the LCD.
4) Multiply the entire equation by the LCD (important note: this doesn't always give us an equivalent equation to the original equation, we must guard against extraneous roots - these are extra answers that may not be part of the solution set).
5) Solve the equation.
6) Check our answers for extraneous roots and to make sure that our answers are not the same as our restrictions.

Ex 1) Solve: $$\frac{x-1}{x-7}=\frac{6}{x-7}$$

$\boxed{x \neq 7}$

LCD: $(x-7)$

$$\frac{(x-7)}{1}\left[\frac{x-1}{x-7}=\frac{6}{x-7}\right]$$

$x-1=6$

$x=7$

Our answer is our restriction

$\therefore$ No solution or $\varnothing$

Ex 2) Solve: $$x + \frac{6}{x} = 5$$

$\boxed{x \neq 0}$

LCD: x

$$x\left[x + \frac{6}{x} = 5\right]$$

$$x^2 + 6 = 5x$$

$$x^2 - 5x + 6 = 0$$

$$(x-2)(x-3) = 0$$

$x = 2$ or $x = 3$

Note: We now have a quadratic equation. Our original equation is linear, we have the potential for extraneous roots. Don't forget to check your answers.

our restriction is $x \neq 0$ so these solutions are allowed in the Domain.

Check our answers:

1) $2+\frac{6}{2}=5$

$2+3=5$

$5=5$ ✓

ok

2) $3+\frac{6}{3}=5$

$3+2=5$

$5=5$ ✓

ok

Ex 3) Solve: $$\frac{3}{x^2-7x+10}+2=\frac{x-4}{x-5}$$

$$\frac{3}{(x-2)(x-5)}+\frac{2}{1}=\frac{(x-4)}{(x-5)}$$

$$\boxed{x\neq 2 \quad x\neq 5}$$

LCD: $(x-2)(x-5)$

$$\frac{(x-2)(x-5)}{1}\left[\frac{3}{(x-2)(x-5)}+\frac{2}{1}=\frac{(x-4)}{(x-5)}\right]$$

$$3 + 2(x-2)(x-5) = (x-4)(x-2)$$

$$3 + 2(x^2 - 7x + 10) = x^2 - 6x + 8$$

$$3 + 2x^2 - 14x + 20 = x^2 - 6x + 8$$

$$x^2 - 8x + 15 = 0$$

$$(x-3)(x-5) = 0$$

$$x = 3 \text{ or } \cancel{x = 5}$$

$$x \neq 5$$

Check:

$$\frac{3}{3^2 - 7(3) + 10} + 2 = \frac{3-4}{3-5}$$

$$\frac{3}{9 - 21 + 10} + 2 = \frac{-1}{-2}$$

$$\frac{3}{-2} + 2 = \frac{1}{2}$$

$$-1\tfrac{1}{2} + 2 = \frac{1}{2}$$

$$\frac{1}{2} = \frac{1}{2} \checkmark$$

Ex 4) An airplane flies 1062 km with the wind. In the same amount of time it can fly 738 km against the wind. The speed of the plane in still air is 200 km/h. Find the speed of the wind.

	Distance =	**Rate x**	**Time**
With Wind	1062	$200+r$	t
Against Wind	738	$200-r$	t

The times are the same:

$$t = \frac{1062}{200+r} \qquad t = \frac{738}{200-r}$$

$$\frac{1062}{200+r} = \frac{738}{200-r} \qquad r \neq -200, \quad r \neq 200$$

LCD: $(200+r)(200-r)$

$$\frac{(200+r)(200-r)}{1}\left[\frac{1062}{200+r} = \frac{738}{200-r}\right]$$

$$1062(200-r) = 738(200+r)$$

$$212{,}400 - 1062r = 147{,}600 + 738r$$

$$64{,}800 = 1800r$$

$$r = 36 \text{ Km/h wind speed}$$