

A square root of a number b is a solution of the equation $x^2 = b$.

Every positive number b has two square roots, denoted by \sqrt{b} and $-\sqrt{b}$

The number 0 has just one square root, 0 itself. Negative numbers don't have *real number* square roots.

The principal square root is its nonnegative square root, denoted by \sqrt{b}

The symbol $\sqrt{\quad}$ is called a radical sign.

An expression written with a radical sign is called a radical expression.

The expression written under the radical sign is called the radicand.

EX 1) Simplify:

$$1a) \sqrt{16} \rightarrow 4$$

$$1b) -\sqrt{16} \rightarrow -4$$

$$1c) \sqrt{\frac{1}{16}} \rightarrow \frac{1}{4}$$

$$1d) \sqrt{0.16} \rightarrow 0.4$$

Ex 2) Find the real roots for each equation, if there are none let us know.

$$2a) x^2 = 25$$

$$\sqrt{x^2} = \sqrt{25}$$

$$|x| = 5$$

$$\oplus x = 5 \quad \square$$

$$\begin{aligned} -x &= 5 \\ x &= -5 \end{aligned}$$

$$x = \pm 5$$

$$2b) x^2 + 64 = 0$$

$$x^2 = -64$$

$$\sqrt{x^2} = \sqrt{-64}$$

$$|x| = \sqrt{-64}$$

No real roots

A cube root of a number b is a solution of the equation $x^3 = b$.

Every positive number b whether positive, negative or zero has exactly one real cube root denoted by $\sqrt[3]{b}$

In the expression $\sqrt[n]{b}$ n is called the root index.

Ex 3) Simplify:

$$3a) \sqrt[3]{27} \rightarrow 3$$

$$3b) \sqrt[3]{-64} \rightarrow -4$$

$$3c) \sqrt[3]{x^{12}} \rightarrow x^4$$

A n th root of a number b is a solution of the equation $x^n = b$.

If n is even and $b > 0$, there are two real n th roots of b .

The principal n th root of b is denoted by $\sqrt[n]{b}$

The other n th root of b is denoted by $-\sqrt[n]{b}$

If n is even and $b = 0$, there is one n th root: $\sqrt[n]{0} = 0$

If n is even and $b < 0$, there is no real n th root of b

If n is odd, there is exactly one real n th root of b , whether b is positive, negative or zero.

Ex 4) Simplify:

$$4a) \sqrt[4]{81} \rightarrow 3$$

$$4b) \sqrt[5]{32} \rightarrow 2$$

$$4c) \sqrt[5]{-32} \rightarrow -2$$

$$4d) \sqrt[17]{-1} \rightarrow -1$$

Properties of Radicals:

1. $(\sqrt[n]{b})^n = b$, since $\sqrt[n]{b}$ satisfies the equation $x^n = b$
2. $\sqrt[n]{b^n} = b$ if n is odd.
3. $\sqrt[n]{b^n} = |b|$ if n is even, because the principal n th root is always nonnegative for even values of n .

Numbers	Squares	Cubes	Fourths	Fifths
1	1	1	1	1
2	4	8	16	32
3	9	27	81	243
4	16	64	256	1024
5	25	125	625	3125
6	36	216	1296	7776
7	49	343	2401	16807
8	64	512	4096	32768
9	81	729	6561	59049
10	100	1000	10000	100000
11	121	1331	14641	161051
12	144	1728	20736	248832
13	169	2197	28561	371293
14	196	2744	38416	537824
15	225	3375	50625	759375
16	256	4096	65536	1048576
17	289	4913	83521	1419857
18	324	5832	104976	1889568
19	361	6859	130321	2476099
20	400	8000	160000	3200000
21	441	9261	194481	4084101

⋮
⋮
25
625