Product and Quotient Properties of Radicals

1. $\sqrt[n]{ab} = \sqrt[n]{a} \bullet \sqrt[n]{b}$

2. $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

Ex 1) Simplify:

1a) $\sqrt{72} = \sqrt{36 \cdot 2}$

$= \sqrt{36}\sqrt{2}$

$= 6\sqrt{2}$

1b) $\sqrt[3]{32} \cdot \sqrt[3]{6} = \sqrt[3]{192}$

$= \sqrt[3]{64}\sqrt[3]{3}$

$= 4\sqrt[3]{3}$

1c) $\sqrt[3]{\frac{81}{125}} = \frac{\sqrt[3]{81}}{\sqrt[3]{125}}$

$= \frac{\sqrt[3]{27} \cdot \sqrt[3]{3}}{5}$

$= \frac{3\sqrt[3]{3}}{5}$

1d) $\frac{\sqrt{72}}{\sqrt{6}} = \sqrt{\frac{72}{6}}$

$= \sqrt{12}$

$= \sqrt{4}\,\sqrt{3}$

$= 2\sqrt{3}$

Ex 2) Simplify:

2a) $\sqrt{3ab^6c^2} = \sqrt{b^6c^2}\,\sqrt{3a}$

$= b^3|c|\sqrt{3a}$

2b) $\sqrt{5x^3} = \sqrt{x^2}\,\sqrt{5x}$

$= |x|\sqrt{5x}$

2c) $\sqrt{9x^2+9y^2} = \sqrt{9}\,\sqrt{x^2+y^2}$

$= 3\sqrt{x^2+y^2}$

A radical expression (containing *n*th roots) is in **simplest radical form** if:

1) no radicand contains a perfect *n*th factor (other than 1) (i.e. no perfect square factors in a square root)
2) no radicand is a fraction
3) no radicals are in the denominator

The process of eliminating the radical (irrational number) from the denominator is called **rationalizing the denominator**.

If there are radicals in the denominator:

1) Find the smallest number that you can multiply the denominator by to get a perfect *n*th factor.
2) Multiply both the numerator and denominator by this number to get an equivalent fraction.
3) Simplify the radical in the denominator (there shouldn't be any radicals left in the denominator).
4) Make sure that your answer is in simplest radical form.

Ex 3) Simplify:

3a) $\sqrt{\frac{7}{5}} = \frac{\sqrt{7}}{\sqrt{5}}$

$= \frac{\sqrt{7}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$

$= \frac{\sqrt{35}}{5}$

3b) $\dfrac{\sqrt[3]{6}}{\sqrt[3]{4}}$

$$= \frac{\sqrt[3]{6}}{\sqrt[3]{4}} \cdot \frac{\sqrt[3]{2}}{\sqrt[3]{2}}$$

What's the smallest number you can multiply 4 by to get a number that is a **perfect cube**?

$$= \frac{\sqrt[3]{12}}{\sqrt[3]{8}}$$

$$= \frac{\sqrt[3]{12}}{2}$$

Numbers	Squares	Cubes	Fourths	Fifths
1	1	1	1	1
2	4	8	16	32
3	9	27	81	243
4	16	64	256	1024
5	25	125	625	3125
6	36	216	1296	7776
7	49	343	2401	16807
8	64	512	4096	32768
9	81	729	6561	59049
10	100	1000	10000	100000
11	121	1331	14641	161051
12	144	1728	20736	248832
13	169	2197	28561	371293
14	196	2744	38416	537824
15	225	3375	50625	759375
16	256	4096	65536	1048576
17	289	4913	83521	1419857
18	324	5832	104976	1889568
19	361	6859	130321	2476099
20	400	8000	160000	3200000
21	441	9261	194481	4084101