Ex 1) Simplify:

1a) $(4\sqrt{3} + 2)(\sqrt{3} - 5\sqrt{2})$

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FOIL $4\sqrt{9} - 20\sqrt{6} + 2\sqrt{3} - 10\sqrt{2}$

$4 \cdot 3 - 20\sqrt{6} + 2\sqrt{3} - 10\sqrt{2}$

$12 - 20\sqrt{6} + 2\sqrt{3} - 10\sqrt{2}$

1b) $(3\sqrt{2} - \sqrt{6})^2$

Remember: $(a-b)^2 = a^2 - 2ab + b^2$

$= (3\sqrt{2})^2 - 2(3\sqrt{2})(\sqrt{6}) + (-\sqrt{6})^2$

$= 9\sqrt{4} - 6\sqrt{12} + \sqrt{36}$

$= 9 \cdot 2 - 6\sqrt{4}\sqrt{3} + 6$

$= 18 - 6 \cdot 2\sqrt{3} + 6$

$= 24 - 12\sqrt{3}$

Reminder: (a+b) and (a-b) are called conjugates. Watch carefully to see what happens when we multiply radical binomials that are conjugates.
Remember the formula: $(a+b)(a-b) = a^2 - b^2$

1c) $\left(2\sqrt{3} + 4\sqrt{5}\right)\left(2\sqrt{3} - 4\sqrt{5}\right)$

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$= 4\sqrt{9} - 16\sqrt{25}$

$= 4 \cdot 3 - 16 \cdot 5$

$= 12 - 80$

$= -68$ No Radicals!

Rationalizing The Denominator revisited:
If the denominator is a binomial that contains a radical, multiply both the numerator and the denominator by the CONJUGATE of the denominator to get an equivalent fraction with a rationalized denominator.

Ex 2) Simplify: $\dfrac{2+\sqrt{3}}{2-\sqrt{3}}$

$$\frac{(2+\sqrt{3})}{(2-\sqrt{3})} \cdot \frac{(2+\sqrt{3})}{(2+\sqrt{3})}$$

$$= \frac{4+2\sqrt{3}+2\sqrt{3}+\sqrt{9}}{4-\sqrt{9}}$$

$$= \frac{4+4\sqrt{3}+3}{4-3}$$

$$= \frac{7+4\sqrt{3}}{1} = 7+4\sqrt{3}$$

Ex 3) If $f(x) = \dfrac{x+1}{x}$, find $f\left(\sqrt{2}+3\right)$

$$f(\sqrt{2}+3) = \frac{(\sqrt{2}+3)+1}{\sqrt{2}+3}$$

$$= \frac{(\sqrt{2}+4)}{(\sqrt{2}+3)} \cdot \frac{(\sqrt{2}-3)}{(\sqrt{2}-3)}$$

$$= \frac{\sqrt{4} - 3\sqrt{2} + 4\sqrt{2} - 12}{\sqrt{4} - 9}$$

$$= \frac{2 + \sqrt{2} - 12}{2 - 9}$$

$$= \frac{-10 + \sqrt{2}}{-7} \cdot \frac{-1}{-1}$$

$$= \frac{10 - \sqrt{2}}{7}$$