

A **radical equation** is an equation in which variables occur in one or more radicands.

Radical Equations	Equations containing radicals, Not Radical Equations
$\sqrt{2x} + 1 = 5$	$\sqrt{2x} = 8$
$\sqrt{x-2} - 7 = -4$	$\sqrt[3]{5} + x = \sqrt[3]{9}$
$\sqrt[3]{x} = -3$	

Theorem: The Principle of Powers

For any natural number n , if $a = b$ is true, then $a^n = b^n$ is true.

Solving Radical Equations:

- 1) Isolate one of the radical terms.
- 2) Use the **Principle of Powers** to raise both sides of an equation to a power that will eliminate a radical.
- 3) If a radical remains, repeat steps 1 and 2 again.
- 4) Check possible solutions for *extraneous roots* and to make sure that they solve the original equation.

Ex 1) Solve:

$$1a) \sqrt{2x-3} = 1$$

$$(\sqrt{2x-3})^2 = (1)^2$$

$$2x - 3 = 1$$

+3 +3

$$\frac{2x}{2} = \frac{4}{2}$$

$$x = 2$$

check:

$$\sqrt{2(2) - 3} \stackrel{?}{=} 1$$

$$\sqrt{4 - 3} \stackrel{?}{=} 1$$

$$\sqrt{1} \stackrel{?}{=} 1$$

$$1 = 1 \quad \checkmark$$

$$1b) 2\sqrt[3]{x} - 1 = 3$$

$$2\sqrt[3]{x} - 1 = 3$$

$$\begin{array}{r} +1 \quad +1 \\ 2\sqrt[3]{x} = 4 \\ \hline 2 \quad \quad 2 \end{array}$$

$$\sqrt[3]{x} = 2$$

$$\left(\sqrt[3]{x}\right)^3 = (2)^3$$

$$x = 8$$

check:

$$2\sqrt[3]{8} - 1 \stackrel{?}{=} 3$$

$$2 \cdot 2 - 1 \stackrel{?}{=} 3$$

$$4 - 1 \stackrel{?}{=} 3$$

$$3 = 3 \checkmark$$

CAUTION

**Extraneous
Roots**



When you raise both sides of a **radical equation** to a power larger than one (**Principal of Powers**), the resulting equation may have a solution that is not a solution of the original equation. This type of solution is called an *extraneous root* (extra root). We must check our answers to verify that all roots satisfy the original equation.

Ex 2) Solve: $3x - 5\sqrt{x} = 2$

$$3x - 5\sqrt{x} = 2$$

$$\begin{array}{r} -3x \qquad \qquad -3x \\ 3x - 5\sqrt{x} = 2 \\ -3x \qquad \qquad -3x \end{array}$$

$$-5\sqrt{x} = 2 - 3x$$

$$5\sqrt{x} = 3x - 2$$

$$(3x - 2)^2 = (5\sqrt{x})^2$$

$$9x^2 - 12x + 4 = 25x$$

~~-25x~~ ~~-25x~~

$$9x^2 - 37x + 4 = 0$$

$$(9x - 1)(x - 4) = 0$$

$$9x - 1 = 0 \quad \text{or} \quad x - 4 = 0$$

$$9x = 1$$

~~$$x = \frac{1}{9}$$~~

~~or~~

$$x = 4$$

check:

$$3\left(\frac{1}{9}\right) - 5\sqrt{\frac{1}{9}} \stackrel{?}{=} 2$$

$$3 - 5 \cdot \frac{1}{3} \stackrel{?}{=} 2$$

$$3 - \frac{5}{3} \stackrel{?}{=} 2$$

$$-1\frac{1}{3} \neq 2$$

$$3 \cdot 4 - 5\sqrt{4} \stackrel{?}{=} 2$$

$$12 - 5 \cdot 2 \stackrel{?}{=} 2$$

$$12 - 10 = 2$$

$$2 = 2 \checkmark$$

Ex 3) Solve $\sqrt{x-3} + \sqrt{x+5} = 4$

$$\sqrt{x-3} + \sqrt{x+5} = 4$$

$$\sqrt{x-3} = 4 - \sqrt{x+5}$$

$$(\sqrt{x-3})^2 = (4 - \sqrt{x+5})^2$$

$$x-3 = (4)^2 - 2(4)(-\sqrt{x+5}) + (-\sqrt{x+5})^2$$

$$x-3 = \underline{16} + 8\sqrt{x+5} + \underline{x+5}$$

$$x-3 = \underline{21} + x + 8\sqrt{x+5}$$

$-x - 21 \quad -21 \quad -x$

$$\frac{-24}{8} = \frac{8\sqrt{x+5}}{8}$$

$$\sqrt{x+5} = -3$$

$$(\sqrt{x+5})^2 = (-3)^2$$

$$x+5 = 9$$

$$\boxed{x = 4}$$

check:

$$\sqrt{4-3} + \sqrt{4+5} \stackrel{?}{=} 4$$

$$\sqrt{1} + \sqrt{9} \stackrel{?}{=} 4$$

$$1 + 3 = 4$$

$$4 = 4 \checkmark$$

The following example is **NOT** a radical equation (since there is **no variable under the radical sign**), but a linear equation that contains a constant or coefficient that is irrational, in other words, an **equation containing radicals**.

Ex 4) Solve: $3x = 2 + x\sqrt{5}$

$$3x = 2 + x\sqrt{5}$$

$$\begin{array}{r} -x\sqrt{5} \\ -x\sqrt{5} \end{array}$$

$$3x - x\sqrt{5} = 2$$

$$x(3 - \sqrt{5}) = 2$$

$$x(3 - \sqrt{5}) = 2$$

$$\frac{x(3 - \sqrt{5})}{(3 - \sqrt{5})} = \frac{2}{(3 - \sqrt{5})}$$

$$x = \frac{2}{(3 - \sqrt{5})} \cdot \frac{(3 + \sqrt{5})}{(3 + \sqrt{5})}$$

$$x = \frac{6 + 2\sqrt{5}}{9 - \sqrt{25}}$$

$$x = \frac{6 + 2\sqrt{5}}{9 - 5}$$

$$x = \frac{6 + 2\sqrt{5}}{4}$$

$$x = \frac{\cancel{2}(3+\sqrt{5})}{\cancel{2} \cdot 2}$$

$$x = \frac{3+\sqrt{5}}{2}$$