A **radical equation** is an equation in which variables occur in one or more radicands.

Radical Equations	Equations containing radicals, Not Radical Equations
$\sqrt{2x}+1=5$	$\sqrt{2}x=8$
$\sqrt{x-2}-7=-4$	$\sqrt[3]{5}+x=\sqrt[3]{9}$
$\sqrt[3]{x}=-3$	

Theorem: The Principle of Powers

For any natural number n, if $a=b$ is true, then $a^n=b^n$ is true.

Solving Radical Equations:

1) Isolate one of the radical terms.
2) Use the **Principle of Powers** to raise both sides of an equation to a power that will eliminate a radical.
3) If a radical remains, repeat steps 1 and 2 again.
4) Check possible solutions for ***extraneous roots*** and to make sure that they solve the original equation.

Ex 1) Solve:

1a) $\sqrt{2x-3}=1$

$$\left(\sqrt{2x-3}\right)^2=(1)^2$$

$$2x-3=1$$
$$+3 \quad +3$$

$$2x=4$$
$$\frac{2x}{2}=\frac{4}{2}$$

$$x=2$$

check:

$$\sqrt{2(2)-3} \stackrel{?}{=} 1$$

$$\sqrt{4-3} \stackrel{?}{=} 1$$

$$\sqrt{1} \stackrel{?}{=} 1$$

$$1=1 \checkmark$$

$$1b)\ 2\sqrt[3]{x}-1=3$$

$$2\sqrt[3]{x}-1=3$$

$$+1 \quad +1$$

$$\frac{2\sqrt[3]{x}}{2}=\frac{4}{2}$$

$$\sqrt[3]{x}=2$$

$$\left(\sqrt[3]{x}\right)^3=(2)^3$$

$$x=8$$

check:

$$2\sqrt[3]{8}-1\stackrel{?}{=}3$$

$$2\cdot 2-1\stackrel{?}{=}3$$

$$4-1\stackrel{?}{=}3$$

$$3=3\ \checkmark$$

When you raise both sides of a **radical equation** to a power larger than one (**Principal of Powers**), the resulting equation may have a solution that is not a solution of the original equation. This type of solution is called an ***extraneous root*** (extra root). We must check our answers to verify that all roots satisfy the original equation.

Ex 2) Solve: $3x - 5\sqrt{x} = 2$

$$3x - 5\sqrt{x} = 2$$

$$-3x \qquad -3x$$

$$-5\sqrt{x} = 2 - 3x$$

$$5\sqrt{x} = 3x - 2$$

$$(3x - 2)^2 = (5\sqrt{x})^2$$

$$9x^2 - 12x + 4 = 25x$$

$$-25x \qquad -25x$$

$$9x^2 - 37x + 4 = 0$$

$$(9x - 1)(1x - 4) = 0$$

$$9x - 1 = 0 \text{ or } x - 4 = 0$$

$$9x = 1 \quad \cancel{\text{or}} \quad x = 4$$

$$\cancel{x = \frac{1}{9}}$$

Check:

$$3\left(\frac{1}{9}\right) - 5\sqrt{\frac{1}{9}} \stackrel{?}{=} 2$$

$$3 - 5 \cdot \frac{1}{3} \stackrel{?}{=} 2$$

$$3 - \frac{5}{3} \stackrel{?}{=} 2$$

$$-1\frac{1}{3} \neq 2$$

$$3 \cdot 4 - 5\sqrt{4} \stackrel{?}{=} 2$$

$$12 - 5 \cdot 2 \stackrel{?}{=} 2$$

$$12 - 10 = 2$$

$$2 = 2 \checkmark$$

Ex 3) Solve $\sqrt{x-3}+\sqrt{x+5}=4$

$$\sqrt{x-3}+\sqrt{x+5}=4$$

$$\sqrt{x-3}=4-\sqrt{x+5}$$

$$\left(\sqrt{x-3}\right)^2=\left(4-\sqrt{x+5}\right)^2$$

$$x-3=(4)^2-2(4)(-\sqrt{x+5})+(-\sqrt{x+5})^2$$

$$x-3=16+8\sqrt{x+5}+x+5$$

$$x-3=21+x+8\sqrt{x+5}$$

$$-x-21 \quad -21 \quad -x$$

$$\frac{-24}{8}=\frac{8\sqrt{x+5}}{8}$$

$$\sqrt{x+5}=-3$$

$$\left(\sqrt{x+5}\right)^2=(-3)^2$$

$$x+5=9$$

$$x=4$$

Check:

$$\sqrt{4-3}+\sqrt{4+5}\stackrel{?}{=}4$$

$$\sqrt{1}+\sqrt{9}\stackrel{?}{=}4$$

$$1+3=4$$

$$4=4\ \checkmark$$

The following example is **NOT a radical equation** (since there is **no variable under the radical sign**), but a linear equation that contains a constant or coefficient that is irrational, in other words, an **equation containing radicals**.

Ex 4) Solve: $3x = 2 + x\sqrt{5}$

$$3x = 2 + x\sqrt{5}$$
$$-x\sqrt{5} \qquad -x\sqrt{5}$$

$$3x - x\sqrt{5} = 2$$

$$x(3-\sqrt{5}) = 2$$

$$\frac{x(3-\sqrt{5})}{(3-\sqrt{5})} = \frac{2}{(3-\sqrt{5})}$$

$$x = \frac{2}{(3-\sqrt{5})} \cdot \frac{(3+\sqrt{5})}{(3+\sqrt{5})}$$

$$x = \frac{6+2\sqrt{5}}{9-\sqrt{25}}$$

$$x = \frac{6+2\sqrt{5}}{9-5}$$

$$x = \frac{6+2\sqrt{5}}{4}$$

$$x = \frac{\cancel{2}(3+\sqrt{5})}{\cancel{2}\cdot 2}$$

$$x = \frac{3+\sqrt{5}}{2}$$