Completeness Property of Real Numbers
Every real number has a decimal representation and every decimal represents a real number.

Remember: a Rational Number is any number that can be expressed as the ratio, or quotient, of two integers (the denominator cannot equal zero).

Ex 1) Find a decimal representation for each rational number:

1a) \( \frac{37}{16} \rightarrow 16\overline{37} \rightarrow 2.3125 \) terminating decimal

1b) \( \frac{19}{22} \rightarrow 22\overline{19} \rightarrow 0.863636363 \ldots \) repeating decimal or \( 0.8\overline{63} \) repeating decimal

The decimal representation of any rational number is either terminating or repeating.
Every terminating or repeating decimal represents a rational number (which can be written as a quotient of two integers \( a/b \) where \( b \) is not equal to 0).
Ex 2) Write each terminating decimal as a common fraction simplified to lowest terms.

2a) \( 2.571 \rightarrow 2 \frac{571}{1000} \)

2b) \( 0.0036 \rightarrow \frac{36}{10\,000} \rightarrow \frac{4.9}{4.2500} \rightarrow \frac{9}{2500} \)

Ex 3) Write each repeating decimal as a common fraction in lowest terms.

3a) \( 0.3\overline{27} \)

Let \( N = 0.3\overline{27} \)

\[ 10N = 3.\overline{27} \]

\[ 1000N = 327.\overline{27} \]

Subtract:

\[ 1000N = 327.\overline{27} \]

\[ - 10N = 3.\overline{27} \]

\[ 990N = 324 \]

\[ N = \frac{324}{990} \]

Get the repeating decimal block by itself.

Strategy: Line up the repeating decimals and subtract.

\[ N = \frac{18}{55} \]
3b) 1.89189
Let \( N = 1.89189 \)

\[ 1000N = 1891.89189 \]

Subtract:

\[ 1000N = 1891.89189 \]
\[ - N = 1.89189 \]

\[ 999N = 1890 \]

\[ N = \frac{1890}{999} \]

\[ N = \frac{70}{37} \]

An irrational number is a real number that is NOT rational. Therefore, the decimal representation is neither terminating nor repeating.

\[ \pi = 3.14159265... \]

\[ \sqrt{3} = 1.7320508... \]
The decimal representation of any **irrational number** is infinite and nonrepeating.
Every **infinite and nonrepeating decimal** represents an **irrational number**.

Ex 4) Classify each real number as either rational or irrational.

4a) \( \sqrt{2} \)

\[ \sqrt{2} = 1.41421356... \quad \text{an infinite, non-repeating decimal} \]

:: \( \sqrt{2} \) is irrational

4b) \( \sqrt[4]{\frac{4}{9}} \rightarrow \frac{\sqrt[4]{4}}{\sqrt[4]{9}} \rightarrow \frac{2}{3} \rightarrow 0.\overline{6} \quad \text{an infinite, repeating decimal.} \)

:: \( \sqrt[4]{\frac{4}{9}} \) is rational

4c) \( 2.030303... \)

\( 2.\overline{03} \) infinite repeating decimal.

:: \( 2.\overline{03} \) is rational
Ex 5) Find a rational number \( r \) and an irrational number \( s \) between 1.51287 and 1.51288.

\[ 1.51287 \]

\[ r = 1.51287\overline{11111111} \ldots \]

\[ s = 1.51287 \overline{010101010101010101} \ldots \]

\[ 1.51288 \]