Completeness Property of Real Numbers

Every real number has a decimal representation and every decimal represents a real number.

Remember: a Rational Number is any number that can be expressed as the ratio, or quotient, of two integers (the denominator cannot equal zero).

Ex 1) Find a decimal representation for each rational number:

1a) $\frac{37}{16} \rightarrow 16\overline{)37} \rightarrow 2.3125$ terminating decimal

1b) $\frac{19}{22} \rightarrow 22\overline{)19} \rightarrow 0.8636363\ldots$ or $0.8\overline{63}$ repeating decimal

The decimal representation of any **rational number** is either *terminating* or *repeating.*

Every *terminating or repeating decimal* represents a **rational number** (which can be written as a quotient of two integers a/b where b is not equal to 0).

Ex 2) Write each terminating decimal as a common fraction simplified to lowers terms.

2a) $2.571 \rightarrow 2\frac{571}{1000}$

2b) $0.0036 \rightarrow \frac{36}{10{,}000} \rightarrow \frac{4 \cdot 9}{4 \cdot 2500} \rightarrow \frac{9}{2500}$

Ex 3) Write each repeating decimal as a common fraction in lowest terms.

3a) $0.3\overline{27}$

Let $N = 0.3\overline{27}$

$10N = 3.\overline{27}$

$1000N = 327.\overline{27}$

Get the repeating decimal block by itself.

Strategy: Line up the repeating decimals and subtract.

subtract:

$$\begin{array}{r} 1000N = 327.\overline{27} \\ -\ 10N = 3.\overline{27} \\ \hline 990N = 324 \end{array}$$

$N = \frac{324}{990}$

$N = \frac{18}{55}$

3b) $1.89\overline{189}$

Let $N = 1.89\overline{189}$

$1000N = 1891.89\overline{189}$

subtract:

$$
\begin{array}{rcr}
1000N & = & 1891.89\overline{189} \\
- \quad N & = & 1.89\overline{189} \\
\hline
999N & = & 1890
\end{array}
$$

$$N = \frac{1890}{999}$$

$$N = \frac{70}{37}$$

An irrational number is a real number that is NOT rational. Therefore, the decimal representation is neither terminating nor repeating.

$\pi = 3.14159265...$

$\sqrt{3} = 1.7320508...$

The decimal representation of any **irrational number** is **infinite and nonrepeating**.
Every **infinite and nonrepeating decimal** represents an **irrational number**.

Ex 4) Classify each real number as either rational or irrational.

4a) $\sqrt{2}$

$\sqrt{2} = 1.41421356\ldots$ an infinite non-repeating decimal

$\therefore \sqrt{2}$ is irrational

4b) $\sqrt{\frac{4}{9}} \rightarrow \frac{\sqrt{4}}{\sqrt{9}} \rightarrow \frac{2}{3} \rightarrow 0.\overline{6}$

an infinite repeating decimal.

$\therefore \sqrt{\frac{4}{9}}$ is rational

4c) $2.030303\ldots$

$2.\overline{03}$ infinite repeating decimal.

$\therefore 2.\overline{03}$ is rational

4d) 2.030030003...

There is a pattern, but this is still an infinite, nonrepeating decimal and is therefore irrational.

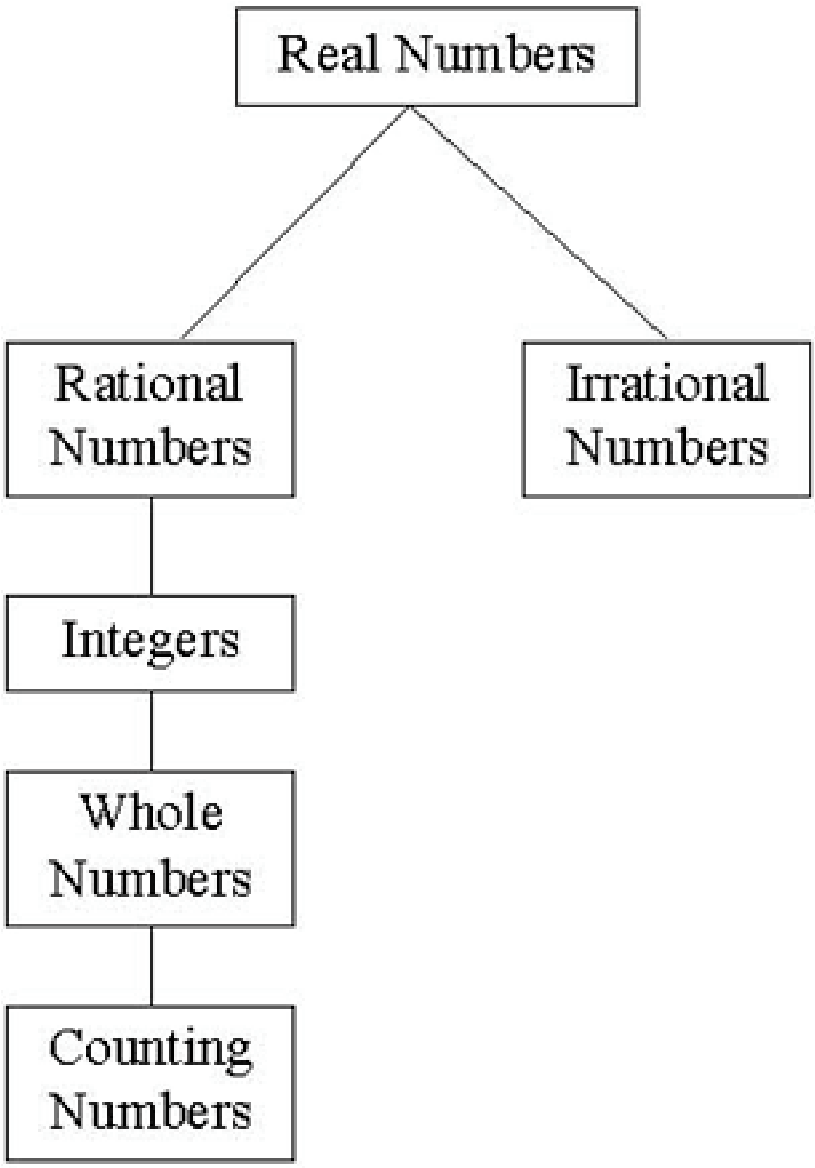
Ex 5) Find a rational number r and an irrational number s between 1.51287 and 1.51288.

1.51287

$\boldsymbol{r} = 1.51287111111111\ldots$

$\boldsymbol{s} = 1.51287010110111011110\ldots$

1.51288


Real Numbers
Rational Numbers
Irrational Numbers
Integers
Whole Numbers
Counting Numbers