This lesson **prepares** the student for the Algebra 2 Standard:
6.0 Students add, subtract, multiply, and divide complex numbers.

Objectives:
The students will:
1. Understand the need for the number i in solving equations.
2. Know the definition of the number i.
3. Know how to rewrite the square root of a negative number in an equivalent form using the number i.
4. Be able to use the number i to simplify square roots of negative numbers.

Throughout history new numbers have been invented to extend the existing number system as necessary to solve equations that were previously unsolvable.
For example, negative numbers were invented to solve an equation like: $x + 3 = 2$.
Likewise, irrational numbers were invented to solve an equation like: $x^2 = 2$.

Now, we need to learn the new numbers necessary to solve an equation like: $x^2 + 1 = 0$, since in the set of real numbers, negative numbers do NOT have square roots.
For example:

$$x^2 + 1 = 0$$

$$x^2 = -1$$

$$\sqrt{x^2} = \sqrt{-1}$$ What number times itself equals -1?

$|x|$ = no real solution

Definition:
The imaginary numbers consist of all numbers bi, where b is a real number and i is the imaginary unit, with the property that $i^2 = -1$ and $i = \sqrt{-1}$

If r is a positive real number, then $\sqrt{-r} = i\sqrt{r}$

Strategy: Factor out the $\sqrt{-1}$ and replace it with i

Example 1, Simplify:

1a) $\sqrt{-5} \rightarrow \sqrt{-1} \cdot \sqrt{5} \rightarrow i\sqrt{5}$

1b) $\sqrt{-25} \rightarrow \sqrt{-1} \cdot \sqrt{25} \rightarrow 5i$

1c) $\sqrt{-50} \rightarrow \sqrt{-1} \cdot \sqrt{25} \cdot \sqrt{2} \rightarrow 5i\sqrt{2}$

Example 2, Simplify:

2a) $\sqrt{-16} - \sqrt{-49}$

$\sqrt{-1} \cdot \sqrt{16} - \sqrt{-1}\sqrt{49}$

$4i - 7i$

$-3i$

2b) $i\sqrt{2} + 3i\sqrt{2}$

$4i\sqrt{2}$

Example 3, Simplify:

3a) $\sqrt{-4} \cdot \sqrt{-25}$

$\sqrt{-1} \cdot \sqrt{4} \cdot \sqrt{-1} \cdot \sqrt{25}$

$2i \cdot 5i$

$10i^2$ Don't forget that $i^2 = -1$

$10(-1)$

-10

Mistake Alert!

Convert to imaginary numbers before multiplying!

$\sqrt{-4} \cdot \sqrt{-25} \neq \sqrt{100}$

$\neq 10$

3b) $i\sqrt{2} \cdot i\sqrt{3}$

$i^2 \sqrt{6}$

$-1\sqrt{6}$

Example 4, Simplify:

4a) $\frac{2}{3i}$

$\frac{2}{3i} \cdot \frac{i}{i}$

$\frac{2i}{3i^2}$

$\frac{2i}{-3} \rightarrow -\frac{2i}{3}$

4b) $\dfrac{6}{\sqrt{-2}}$

$\dfrac{6}{i\sqrt{2}}$

$\dfrac{6}{i\sqrt{2}} \cdot \dfrac{i\sqrt{2}}{i\sqrt{2}}$

$\dfrac{6i\sqrt{2}}{2i^2} \rightarrow \dfrac{6i\sqrt{2}}{-2} \rightarrow -3i\sqrt{2}$

Example 5, Simplify:

5a) $\sqrt{-9x^2} + \sqrt{-x^2}$

$\sqrt{-1} \cdot \sqrt{9} \cdot \sqrt{x^2} + \sqrt{-1} \cdot \sqrt{x^2}$

$3ix + ix$

$4ix$

5b) $\sqrt{-6y} \cdot \sqrt{-2y}$

$\sqrt{-1} \cdot \sqrt{6y} \cdot \sqrt{-1} \cdot \sqrt{2y}$

$i \cdot \sqrt{6y} \cdot i \cdot \sqrt{2y}$

$i^2 \cdot \sqrt{12y^2}$

$-1 \sqrt{4y^2} \cdot \sqrt{3}$

$-2|y|\sqrt{3}$

Example 6, Solve:

6) $2x^2 + 19 = 3$

$\quad\;\; -19 \quad -19$

$\dfrac{2x^2}{2} = \dfrac{-16}{2}$

$x^2 = -8$

$\sqrt{x^2} = \sqrt{-8}$

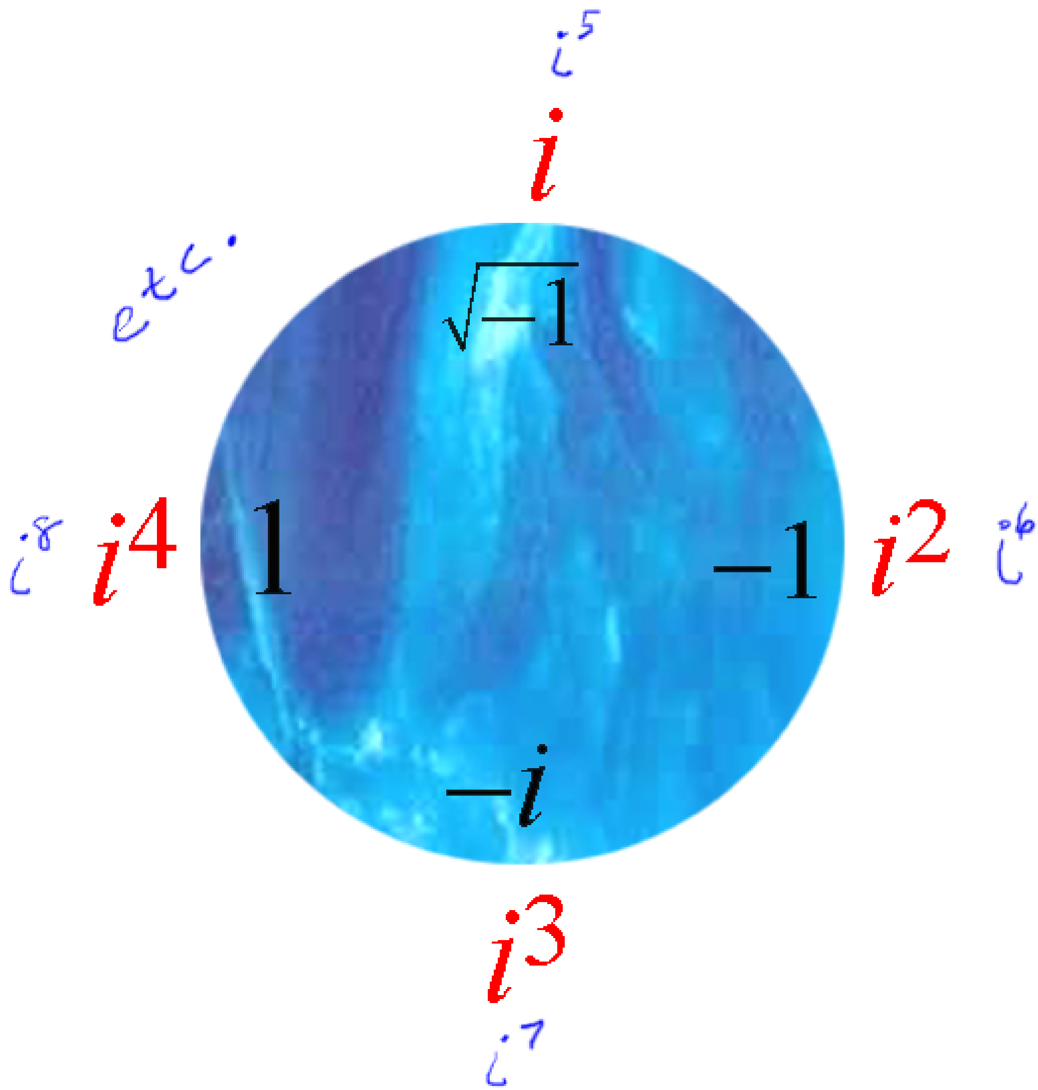
$$|x| = \sqrt{-1} \cdot \sqrt{4} \cdot \sqrt{2}$$

$$|x| = 2i\sqrt{2}$$

$$x = \pm 2i\sqrt{2}$$

The powers of ***i*** and the ***"i-cycle"***

$i = \sqrt{-1}$ $\quad i^2 = -1$ $\quad i^3 = -i$ $\quad i^4 = 1$



Example 7, Simplify:

7a) $i^{10} \rightarrow 4\overline{)10}$, $\cancel{2}$, -8, 2 $\rightarrow i^2 \rightarrow -1$

7b) $i^{27} \rightarrow 4\overline{)27}$, $\cancel{6}$, -24, 3 $\rightarrow i^3 \rightarrow -i$

7c) $i^{124} \rightarrow 4\overline{)124}$, $\cancel{31}$, -12, 4, -4, 0 $\rightarrow i^0 \rightarrow 1$

When we divide two integers, ignore the quotient and keep the remainder, we are doing ***modular*** or "clock arithmetic". The remainder is called the ***modulus***. In a clock there are 12 numbers in the cycle, while the powers of ***i*** have 4 numbers in their cycle of exponents.