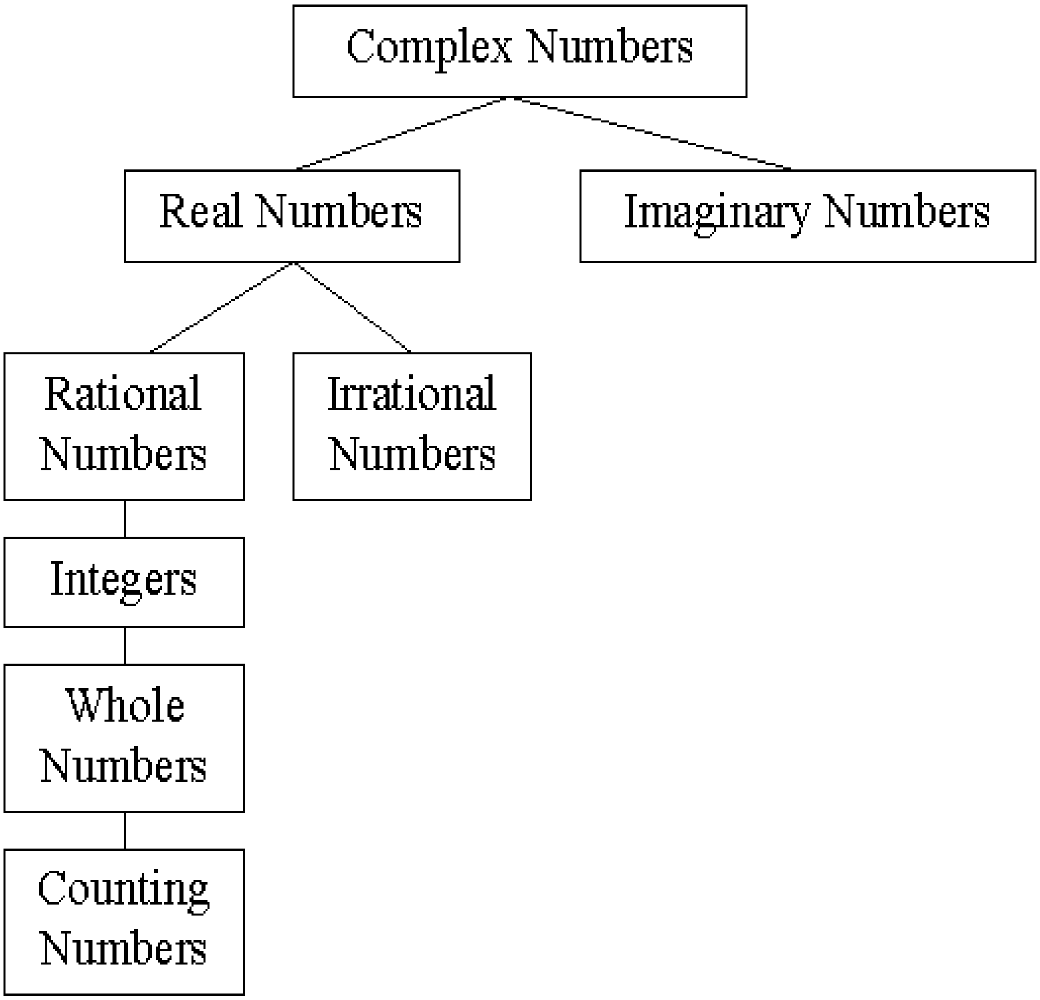
Algebra 2 6-8 The Complex Numbers Page 292

Alg 2 Standard 6.0: Students add, subtract, multiply, and divide complex numbers.
Alg 2 Standard 5.0: Students demonstrate knowledge of how real and complex numbers are related both arithmetically and graphically. In particular, they can plot complex numbers as points in the plane.

Objectives: The student will be able to:
1. Add complex numbers
2. Subtract complex numbers
3. Multiply complex numbers
4. Divide complex numbers
5. Graph complex numbers in the complex plane.
6. Identify how complex numbers are related to the other number systems.

The **real numbers** and the **imaginary numbers** together form the set of **complex numbers**.
A **complex number** is a number in the form of $a + bi$, where a and b are real numbers. We call the number a the **real part** of the complex number and b the **imaginary part**.

The set of the complex numbers can be illustrated as follows:



Equality of Complex Numbers

$a + bi = c + di$ **if and only if** $a = c$ and $b = d$

Sum of Complex Numbers

$(a + bi) + (c + di) = (a + c) + (b + d)i$

Product of Complex Numbers

$(a + bi)\,(c + di) = (ac - bd) + (ad + bc)i$

Example 1 Simplify:

1a) $(3+4i) + (-2-6i)$

$= (3+-2) + (4+-6)i$

$1-2i$

1b) $(7+2i) - (3-6i)$

$(7+2i) + (-3+6i)$

$(7+-3) + (2+6)i$

$4+8i$

Example 2 Simplify:

2a) $(2+3i)(5-6i)$

$= 10 - 12i + 15i - 18i^2$

$= 10 + 3i - 18(-1)$

$= 28 + 3i$

2b) $(2-4i)^2$

$= 2^2 + 2(2)(-4i) + (-4i)^2$

$= 4 - 16i + 16i^2$

$= 4 - 16i + 16(-1)$

$= -12 - 16i$

2c) $(2+3i)(2-3i)$

$4 - 9i^2$

$4 - 9(-1)$

$4 + 9$

13

Example 3 Simplify:

3) $\frac{2-i}{2-3i}$

$\frac{2-i}{2-3i} \cdot \frac{2+3i}{2+3i}$

$\frac{4+6i-2i-3i^2}{4-9i^2}$

$\frac{4+4i-3(-1)}{4-9(-1)}$

$\frac{7+4i}{13}$

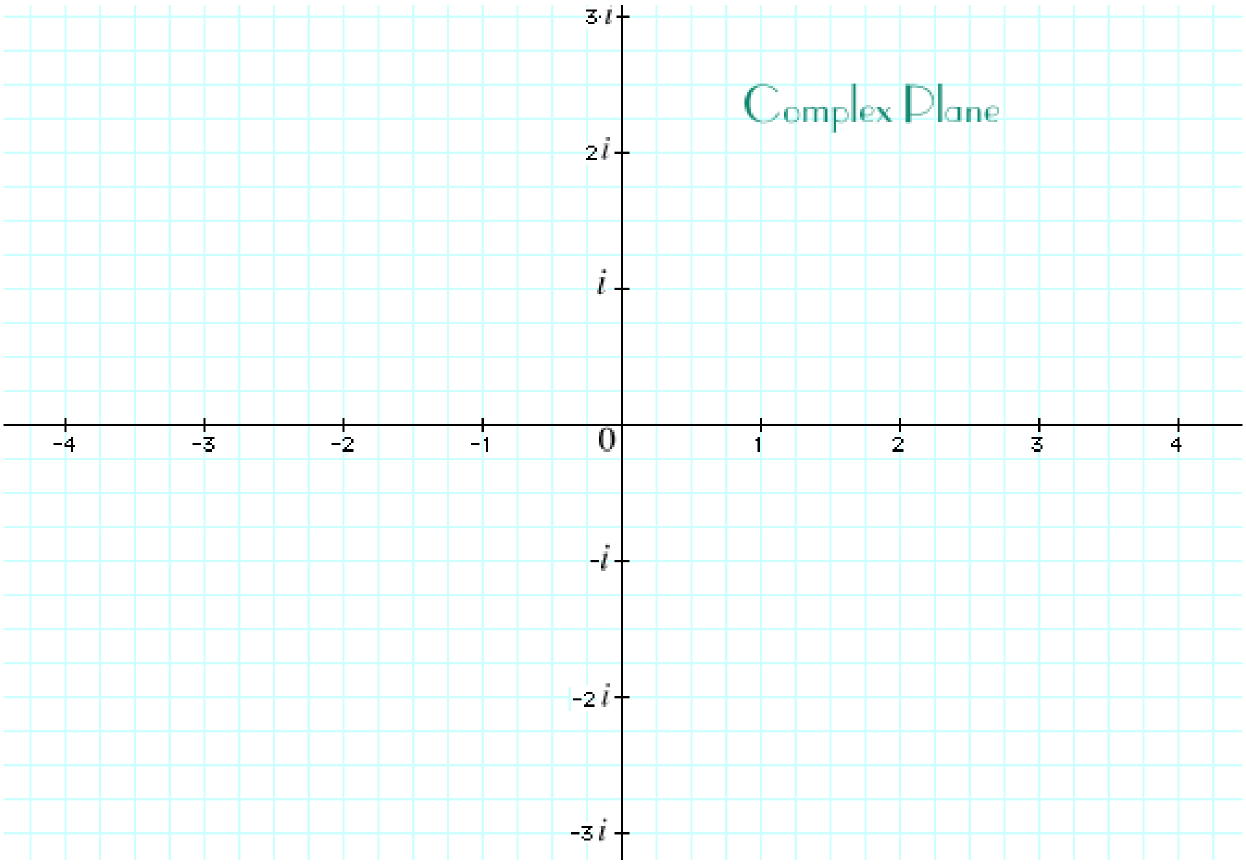
Example 4 Find the reciprocal of *2 - i*

$$\frac{1}{2-i}$$

$$\frac{1}{2-i} \cdot \frac{2+i}{2+i}$$

$$\frac{2+i}{4-i^2} \rightarrow \frac{2+i}{4-(-1)} \rightarrow \boxed{\frac{2+i}{5}}$$

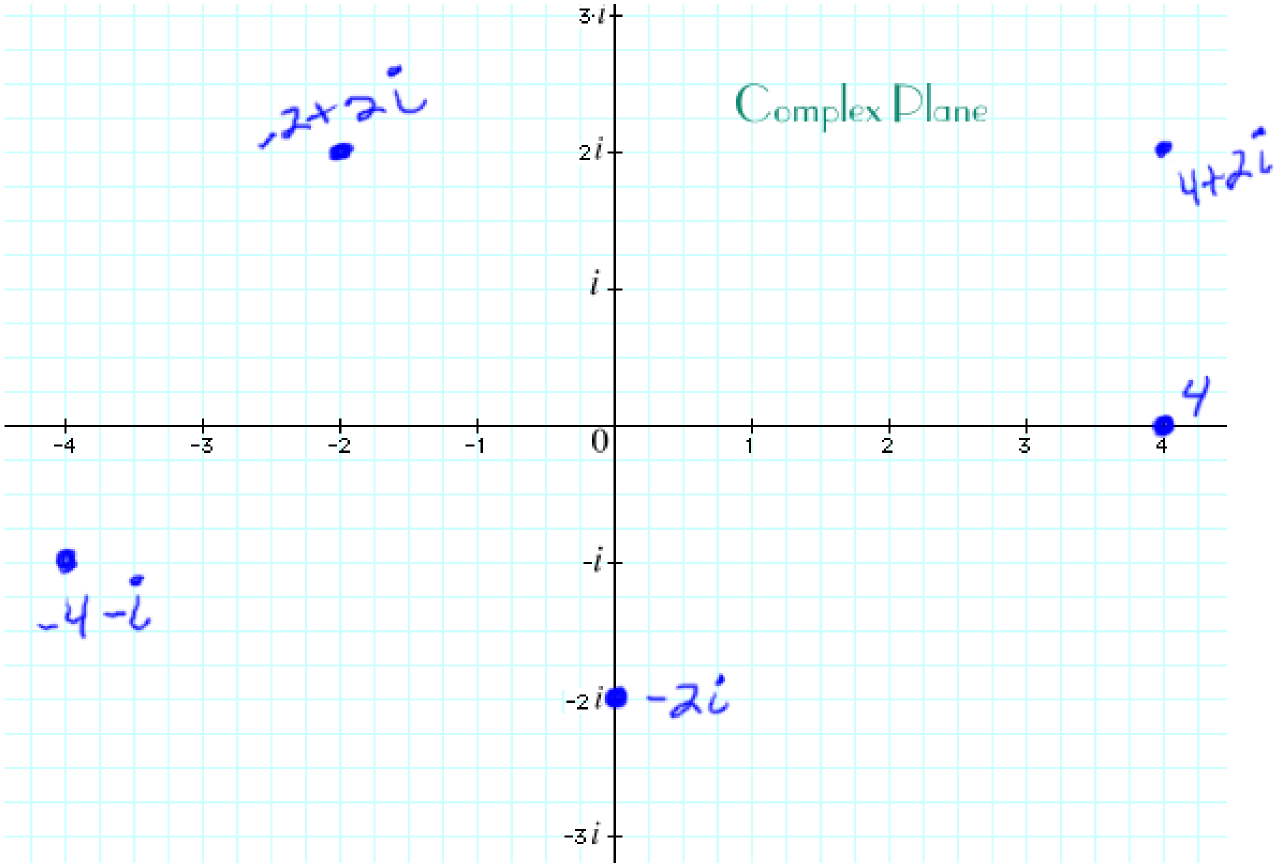
We can represent complex numbers in a coorinate plane by allowing $x + yi$ to correspond to the point (x, y). We call this plane the **Complex Plane**. The horizontal axis is called the **real axis** and the vertical axis is called the **imaginary axis**.



http://www.clarku.edu/~djoyce/complex

Real numbers are considered special cases of complex numbers; they are the numbers $x + yi$ when y is 0. We graph these numbers on the real axis (horizontal axis). The numbers on the imaginary axis (vertical axis) are called imaginary numbers just as we studied in a previous lesson and are represented as $x + yi$ when x is 0.

Example 5 Graph the following complex numbers on the complex plane:
4, -2*i*, 4 + 2*i*, -4 - *i*, and -2 + 2*i*.



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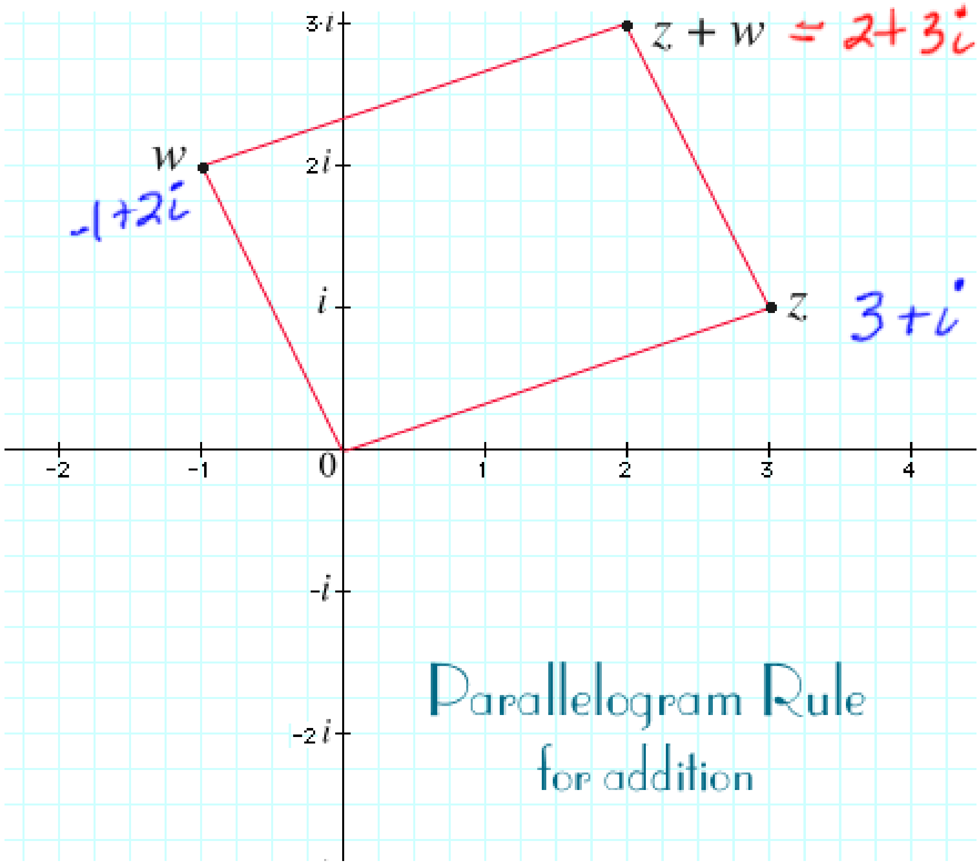
Graphing gives us a Geometric way to represent complex numbers. Whereas, in the beginning of this lesson we represented them Algebraically as in the expression $x + yi$.

Geometrically, addition can be represented graphically on the complex plane. Consider the example: $(3 + i) + (-1 + 2i)$. The complex number $z = 3 + i$ is located 3 units to the right of the imaginary axis and 1 unit above the real axis, while $w = -1 + 2i$ is located 1 unit left and 2 units up.
So the sum $z + w = 2 + 3i$ is 2 units right and 3 units up.



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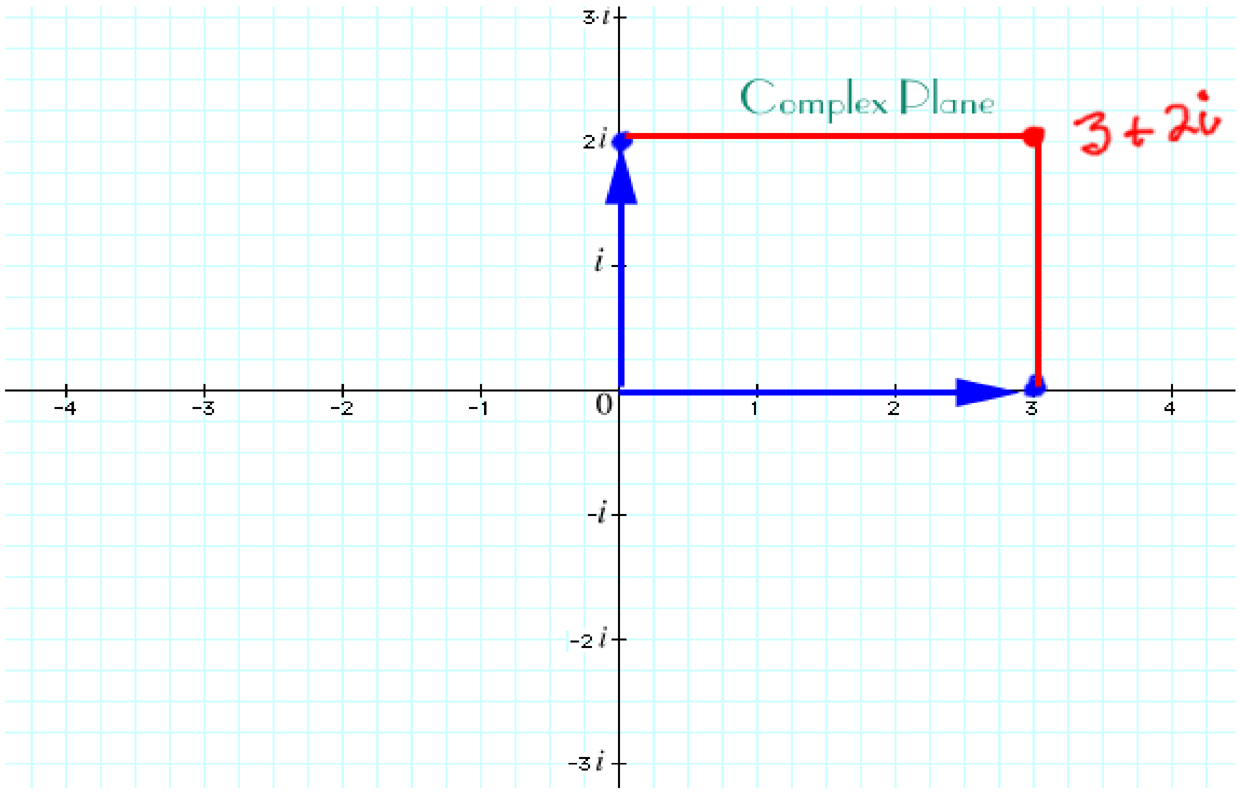
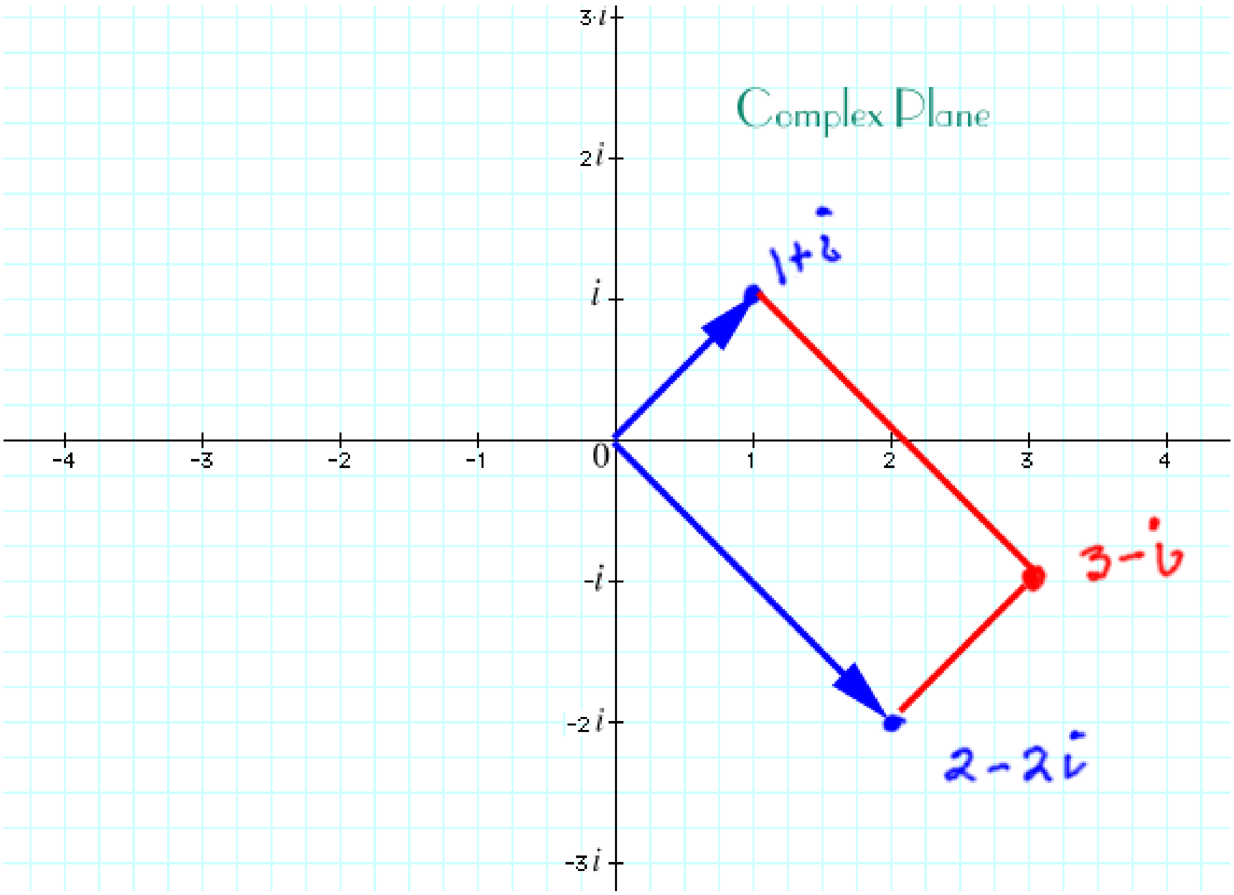
Parallelogram Rule. Note in the last example that the four complex numbers 0, z = 3 + i, w = –1 + 2i, and z + w = 2 + 3i are the corners of a parallelogram. This is generally true.
To find where in the Complex Plane **C** the sum z + w, of two complex numbers, z and w is located, plot z and w (the two complex numbers), draw lines from 0 to each of them, and complete the parallelogram.
The fourth vertex will be the sum of the two complex numbers, z + w.

Example 6: Graph the following complex numbers and use the Parallelogram Rule to find and graph the sum. Add 3 and 2***i*.**


Complex Plane
3+2i
3·i
2i
i
-i
-2i
-3i
-4
-3
-2
-1
0
1
2
3
4

Example 7: Graph the following complex numbers and use the Parallelogram Rule to find and graph the sum. Graph and add: (2 - 2***i***) + (1 + ***i***).



Thanks to David Joyce, Clark University, for the information on graphing complex numbers. This information can be found at:
http://www.clarku.edu/~djoyce/complex/