Sometimes we don't need to know the actual solutions to quadratic equations, we just need to know what type of solutions they are.
Look at the examples below:

<u>Ex 1</u> $x^2 - 5x + 6 = 0$

$a = 1 \quad b = -5 \quad c = 6$

$$x = \frac{+5 \pm \sqrt{(-5)^2 - 4(1)(6)}}{2(1)}$$

$$x = \frac{5 \pm \sqrt{25 - 24}}{2}$$

$$x = \frac{5 \pm \sqrt{1}}{2} \leftarrow \text{a positive number}$$

$$x = \frac{5+1}{2} \; ; \; x = \frac{5-1}{2}$$

$x = 3$ or $x = 2$

2 real roots

Ex2 $x^2 - 4x + 4 = 0$

$a = 1 \quad b = -4 \quad c = 4$

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(4)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{16 - 16}}{2}$$

$$x = \frac{4 \pm \sqrt{0}}{2}$$ ← radicand is zero

$$x = \frac{4+0}{2} \; ; \; x = \frac{4-0}{2}$$

$x = 2$ or $x = 2$

$x = 2$ double root

One real root

Ex3 $x^2+x+1=0$

$a=1 \quad b=1 \quad c=1$

$$x=\frac{-1\pm\sqrt{1^2-4(1)(1)}}{2(1)}$$

$$x=\frac{-1\pm\sqrt{1-4}}{2}$$

$$x=\frac{-1\pm\sqrt{-3}}{2} \leftarrow \text{radicand is negative}$$

$$x=\frac{-1\pm i\sqrt{3}}{2}$$

2 complex roots

Notice the following observations about our solutions:

1) We have 2 real roots when the radicand was Positive
2) We have 1 real (double) root when the radicand was Zero
3) We have 2 complex roots when the radicand was Negative

The radicand of the quadratic formula is: b^2-4ac

we call this the Discriminant

The Discriminant tells us what "types" of answers we will get when solving quadratic equations. Sometimes, we don't need to know the actual solutions, just what type of solution it is.

When we want to know what type of solutions, we say, "Determine the nature of the solutions"...

All we need to do is calculate b^2-4ac to see if its positive, zero, or negative.

Determine the nature of solutions:

2a) $3x^2-7x+5=0$

$a=3 \quad b=-7 \quad c=5$

$(-7)^2-4(3)(5)$

$49-60=-11$

2 complex roots

2b) $2x^2-13x+15=0$

$a=2 \quad b=-13 \quad c=15$

$(-13)^2-4(2)(15)$

$169-120=49$

2 real roots

(both are rational, since $\sqrt{49}=7$)

Ex 2c) $x^2 + 2\sqrt{3}x - 1 = 0$

$a = 1 \quad b = 2\sqrt{3} \quad c = -1$

$(2\sqrt{3})^2 - 4(1)(-1)$

$12 + 4 = 16$

2 real roots

(both are rational, since $\sqrt{16} = 4$)